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THE PRE-SCHOOL CHILD'S CONCEPT OF NUMBER

A Thesis

Submitted to

the Faculty of Graduate Studies

University of Alberta

Department of Elementary Education

In Partial Fulfillment

of the Requirements for the Degree

Master of Education

by

Alec T. Brace

September 1963

ACKNOWLEDGEMENTS

The writer expresses his gratitude to Dr. L. D. Nelson, chairman of his thesis committee for guidance and helpful criticism during the development of this thesis. Appreciation is also extended to Dr. S. Hunka and Dr. W. Worth for their help and thoughtful suggestions. Thanks are also due to professor E. E. Andrews who was a former member of his committee.

Special thanks are due to Mrs. M. Seeger, Director of Playschools for Edmonton, and to the teachers of the playschool classes in which this study was undertaken, without whose kind assistance and excellent cooperation this study would have been impossible. The writer gratefully acknowledges the encouragement of his wife and the help of his daughter, Lorna and his son, Calvin.

Finally, gratitude is also expressed to Mrs. Ivy Carter, who in addition to the typing, also undertook other responsibilities in connection with the final preparation of this thesis.

ABSTRACT

The primary purpose of this study was to determine to what extent pre-school children have developed some of the basic concepts of number. The study concerned itself with the following specific concepts:--

1. Rational counting
2. Comparisons--group relations
3. The conservation of number
4. Cardinal number
5. Ordinal number
6. Place value as applied to the decimal system of notation

In addition the study sought to clarify the nature of the relationships between the child's knowledge of basic number concepts and ability to count, sex, influence of older siblings, socio-economic status or environment, and chronological age.

A test made up of six subtests, each designed to measure a particular concept, was constructed and administered individually to a sample of one hundred twenty-four pre-school children. These children ranged in age from five years four months to six years five months and had all been attending playschools in the city of Edmonton, Alberta, Canada, for a period of six months prior to the time the tests were administered in March and April 1963.

The results of the data thus collected were analysed. A summary of the children's performance on each of the various subtests was made. The responses of these children to each of the items dealing with the basic concepts of number were factor analysed in an attempt to determine

what factors were being measured and to what extent the various subtests were independent of each other. Further analysis of factor scores, based on the items defining the various factors, was made in an attempt to clarify the relationships noted earlier.

Perhaps the most important finding of this investigation was the tremendous gap between the child's ability to count and his knowledge of the basic ideas of number. The relationship of counting to knowledge of cardinal number and the conservation of number was found to decrease with the age of the child, being less significant in older pre-school children. That of counting to the child's concept of ordinal number increased with the age of the child. Sex and the influences of older siblings were not found to be contributing to the child's understanding of the basic concepts. On the other hand, chronological age and environment were found to contribute significantly to his knowledge of the number concepts.

On the basis of the findings of this study the following conclusions are drawn:--

1. The child's ability to count is not a reliable criterion of the extent to which he has developed a knowledge of the true concept of number.

2. A thorough knowledge of cardinal number is necessary before the child can have real facility with ordinal number and before he can really appreciate the significance of the counting process.

3. A thorough knowledge of the concepts of cardinal number, ordinal number and rational counting is necessary before the child can understand the structure of our numeration system.

4. Sex and the influences of older siblings do not contribute to the child's understanding of the basic ideas of number.

5. Chronological age and environment are factors which contribute significantly to the child's concept of number.

This study has many implications for educational practice. Some of these implications are presented together with some suggestions for further research.

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CHAPTER I

THE PROBLEM AND DEFINITIONS OF TERMS USED

The growing awareness of the importance of mathematics in our society and the close scrutiny that mathematics programs are now undergoing have resulted in an increased concern about the arithmetic programs of the elementary grades. The result is that the emphasis has been shifted from the development of skill in computation to the development of the basic concepts of number.

A few simple ideas provide the foundation upon which all elementary school mathematics is built. These fundamental ideas include the concept of one-to-one correspondence, the concept of cardinal number, the concept of ordinal number, an understanding of operations and relations as applied to the number system and a thorough knowledge of place value as used in the decimal system of notation.

Many of the newer programs in mathematics for beginning school children are based on the assumption that the young child's knowledge of even the simplest concepts of number may be inadequate. They, therefore, present a sequential development of the concept of number, starting with very primitive ideas, to enable the child to broaden and deepen his learning in a systematic way from the very outset. Formal work with number symbols is undertaken only after these basic concepts have been firmly established.

This approach to the teaching of number is new, and although skill in computation is still regarded as an important aspect of mathematics in the elementary school, this particular aspect has been

relegated to a place of secondary importance. The idea seems to be that once the child has acquired a thorough knowledge of the fundamental concepts of number, computational skills, which received so much emphasis in traditional programs, can be developed meaningfully and with much less difficulty than was the case formerly.

I. THE PROBLEM

Statement of the problem. Is the assumption on which modern programs are based a valid one? Or is it possible that the young child's experiences in his pre-school years have enabled him to develop some or all of the concepts presently being developed systematically in the newer programs? The purpose of this study is to determine to what extent some important aspects of the basic concepts of number have developed in pre-school children. The aspects with which this study will be particularly concerned are, one-to-one correspondence, the invariance of number, cardinal number, ordinal number, and place value as related to the decimal system of notation.

Importance of the study. Learning is a continuous, developmental process. It proceeds gradually from one level of development to another without definable breaks in the process. The child builds new concepts and skills upon those he has already mastered. The teacher must see that the sequence of her instructional activities is based on a meaningful continuity of learning experiences. Violation of this principle will leave gaps in the child's understanding and may adversely affect his progress in later grades. Too much of what passes for learning may

be sheer verbalism, that is, memorization without understanding, and in no subject is this more common than in mathematics. All too frequently the child is expected to memorize number facts and processes that are meaningless to him. Children frequently find themselves, as De May points out, in a "wilderness of unknown symbols and meaningless names."[1]

If healthy attitudes toward mathematics are to be created and children are to be spared the frustration that comes from repeated failure, then instruction must begin at the stage which the child has reached in his number knowledge and proceed systematically and sequentially from there. The stage which the child has reached must be determined on the basis of sound experimental evidence.

Many studies have attempted to determine the young child's number knowledge by making inventories of his verbalizations of the number names. Few attempts have been made, however, to determine the relationship between the child's apparent knowledge of number as revealed by his verbalizations and his actual understanding of the underlying concepts involved. This study will therefore attempt to determine to what extent some basic concepts have developed.

II. LIMITATIONS OF THE STUDY

No attempt will be made to exhaust all facets of the child's number knowledge and related concepts. This study will be confined to some of those aspects considered to be fundamental to the development

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[1]Quoted in Robert Lee Morton, Teaching Arithmetic in the Elementary School, Vol. I, New York: Silver Burdett Co., 1958, p. 5.

of the concept of number with primary school children as noted earlier on page 2.

Further restrictions will include limiting the study to pre-school children in an urban area. It will also be confined to those pre-school children who are attending playschools preparatory to entering regular school in September, 1963. Thus the age range of children involved in the study will be approximately five to six and a half years, since to be eligible for enrollment in playschool, in the area in which this investigation is being undertaken, a child must be five years of age on or before December 31 of the year in which he enrolls. No attempt will be made to equalize the number in each age group within this age range.

It will be assumed that the sample selected is representative of all playschool classes in the area under investigation and all generalizations will be confined to the particular population in question.

Any interpretation of the findings of this study must be undertaken with these limitations in mind.

III. DEFINITION OF TERMS

The following definitions will serve to point out the context in which a number of otherwise common terms will be used throughout this study.

Concepts. A concept may be defined as a generalization about related data. It arises out of perceptions, out of actual acquaintance with objects and situations, and through undergoing experiences and engaging in actions of various kinds. The Concise Oxford Dictionary

defines a concept as "the idea of a class of objects," and conceptualism as the "doctrine that the mind can form ideas corresponding to abstract terms."

The concept of number. Number concepts are generalizations about the relationships that exist between certain kinds of data. They are really the basic ideas underlying number, divorced from objects, actions or number symbols, and must exist in the child's mind.

Rote counting. This is the process of simply repeating the number names in sequence without attempting to attach meaning to them.

Rational counting. This is the process by which objects are enumerated. It consists of making a one-to-one correspondence of the number names with the objects in a set and then assigning a cardinal number to the set.

One-to-one correspondence. When two sets of objects can be matched one-to-one, they are said to be in one-to-one correspondence. The objects may be alike or totally dissimilar. If the one-to-one matching can be carried out, then it can be said that there are as many objects in one group as in the other. This is actually a more immature process than rational counting, since the decision as to the equality of the two sets is arrived at without any reference to the number of objects in either set.

Cardinal number. The cardinal property of number is used to refer to the total group value of a set of objects. When objects are enumerated, it is the cardinal number that is used to refer to the total collection. For example, the cardinal "seven," refers to any collection

of seven objects, which can be counted to form a total which remains the same total regardless of the size, shape, or arrangement of the objects comprising the group. It is an abstract concept and is used to refer to the common property of "seven-ness" in any number of groups of seven objects which may or may not have other common properties.

Ordinal number. The ordinal property of number refers to the rank number of a unit in a series. It is the positional value such as second, third or fourth object in a series.

Conservation. This term will be used to refer to the invariance of a given number, that is, understanding that a given total remains the same regardless of the arrangements or rearrangements of its parts. For example in comparing two equal groups of blocks, the child who believes that there are more blocks in one group when they are spread out than when they are matched in one-to-one correspondence with the other group has not yet acquired conservation of number. His concept of total quantity is being governed by his perceptions and consequently, because one group takes up more space than the other, he believes the number to be greater.

Place value. In this study the term "place value" will be used to refer to the fact that the value assigned to a digit depends upon the position of the digit in the numerals used to represent a number.

Operations. Throughout this study, the term "operations" will be used to refer to the binary operation of addition as applied to number.

Relations. This term refers to the relationship between groups. For example, one group of objects may be referred to as being, "equal

to," "greater than," or "less than" another group. These terms denote the relation of one group to the other.

Playschool. Throughout this study, the term "playschool" will be used to refer to an establishment where boys and girls engage in games and activities and are provided with an opportunity to express themselves through conversation, movement, art, and music. It is primarily a kind of socialized experience and no formal instruction is offered in any subject field. Children normally attend for three two-hour sessions each week for a period of seven months.

Pre-school children. This term will be used to refer to those children who are between the ages of five and six and a half years and who are enrolled in a playschool preparatory to enrollment in regular school.

IV. HYPOTHESES TO BE INVESTIGATED

This study will attempt to investigate the following hypotheses:--

Hypothesis 1.

There is no significant relationship between the child's verbalizations of the number names and his concept of the underlying ideas involved.

Hypothesis 2.

There is no significant difference between the number concepts of boys and girls.

Hypothesis 3.

There is no significant difference in the number concepts of

children of high socio-economic status and those of low socio-economic status.

Hypothesis 4.

There is no significant difference in the number concepts of children with brothers and sisters attending school and those with no brothers and sisters attending school.

Hypothesis 5.

There is no significant difference in the number concepts of children in each of the age groups, five to five and a half years, five and a half to six years, and six to six and a half years.

V. OVERVIEW

Chapter II will be devoted to a review of the literature relative to the child's knowledge of number and the development of the concept of number. Chapter III will contain an outline of the procedures to be followed in conducting the study and the statistical procedures to be used in analysing the results. Chapter IV will contain a report of the findings of this investigation, while in the final chapter a summary of the findings will be presented, together with some recommendations and suggestions for further research.

CHAPTER II

A REVIEW OF RELATED LITERATURE

The development of the concept of number is related to, but can be regarded as distinct from, the child's knowledge of number concepts. One involves the process, the other the product. Studies in the young child's concept of number can therefore be divided into two distinct categories (1) studies related to the child's knowledge of number concepts and (2) studies which are concerned with the development of these number concepts. Most studies have been concerned with the former but in recent years the concern over the way concepts develop in children has led to some studies in this area.

I. STUDIES RELATED TO THE CHILD'S KNOWLEDGE OF NUMBER

During the first half of the 20th century Thorndike's stimulus-response explanation of learning was the dominant theory. An application of this theory to the teaching of arithmetic led to a great dependence on drill. Much emphasis was placed on skill in computation, rather than on the comprehension of the basic concepts involved. It was believed that by manipulating symbols the child would eventually reach the stage where he would be able to apply these skills. Some concepts of number arose in the minds of many children whose teachers used this approach, but from the simplest situations involving small numbers there quickly followed much learning by rote, blind obedience to rules and much oral and written practice. It was on this type of

theory that much of the earlier research in number was based.

But number is an abstract concept entirely independent of the words or symbols used to represent it and completely divorced from the physical attributes of the objects involved. When the child's number knowledge is determined by his responses to questions on number put to him by an experimenter, there is some doubt as to how much of it reveals a true understanding of number and how much consists of merely repeating number names which have no real significance. Many of the earlier studies were conducted in the manner described and caution must be exercised in interpreting the findings.

In spite of the limitations of these studies and of the theory on which they were based, they did represent a serious attempt to determine the young child's knowledge of number. Some of these studies will therefore be reported. The reader should note, however, that in these studies, what is being interpreted as number knowledge is really the child's verbalizations of number rather than a knowledge of the underlying concepts involved.

Number Knowledge of pre-school children. Perhaps the most systematic study of this type was that of Coward (1940) in which a battery of tests from previous studies in number was administered individually to twenty-five girls and twenty-five boys at each of four age levels from three through six years. The tests were designed to determine the child's number ability in such areas as counting by one's and two's, counting objects, producing numbers, number naming, number combinations and simple problems. The results can be summarized as follows:--

1. At no age can all children count without initial help; a great increase in the ability to count appears at the six year level; performance on rote counting is superior to that on object counting.

2. Figures (presumably symbols) are scarcely recognized until the age of six years.

3. Children appear to learn the number names in serial order, but the performance on a given number may not be the same from one type of test to another.

4. Number finding, distinguishing numbers, producing numbers, and number naming rank from easiest to most difficult in that order at all ages; however they follow the same trend of increase with age.

5. Concepts of comparative degree are well understood at all ages.

6. Concepts of measure are little understood until the age of five years.

7. Coins can be identified at all ages but few children can handle money concepts in simple problems.

8. There is slight ability to combine numbers by simple addition until the age of six years.

9. Children have little knowledge of fractions until the age of six years.

10. The ability to solve problems increases steadily with age.

Coward also found that girls were superior to boys in all concepts and at all age levels except four years. The number ability of children of high socio-economic status was consistently superior to that of children of low socio-economic status.

McLaughlin (1935) made an earlier summary of the number knowledge

of pre-school children and concluded that memorization of the number names as used in rote counting proceeds far in advance of the ability to use numbers in enumerating the objects in a series.

From a survey conducted at the Laboratory School of Southern Illinois University, Mott (1945) concluded that young children have considerable knowledge of number. However there is some question as to what is meant by knowledge of number. Her sample consisted of two groups of children between four and five years of age, some of whom had had kindergarten training while the others had not. She tested the children's ability in such areas as rote counting, rational counting, repeating a series of numbers, selecting a given number of cubes from a group, addition and subtraction combinations, recognizing number symbols and writing number symbols. On the basis of this study she concluded that (1) at least ninety per cent of the four and five year olds have developed the concepts for the first ten ordinals (2) about forty-six per cent could perform simple addition and subtraction (3) kindergarten experience facilitates the development of the number concepts.

MacLatchy (1951) developed an interview test for pre-school children's familiarity with measurement. Using this instrument with seventy-three children ranging in age from three to five years, she reported consistent growth in the child's familiarity with measurement.

Davis (1951) employed the test devised by MacLatchy with twenty-three four-year-olds enrolled in a nursery school and twenty-nine five-year-olds enrolled in a campus demonstration school. He concluded that the five-year-old's knowledge of measurement was superior to that of the

four-year-olds, but while there were marked differences in familiarity with measurement between the responses of the four- and five-year-old children, the older children had not achieved mastery of the ideas.

Martin (1951) investigated the quantitative concepts of fifteen boys and fifteen girls at each age level from three to seven years. He used two types of tests (1) a test of quantitative expression and (2) a test of number ability. The former consisted of twenty cards with numerical content such as three flags or a group of houses and was designed to measure productivity, that is the total number of words used. The latter test of number ability was the one used by Coward in a study reported earlier. Martin's findings substantiate those of previous studies, with one exception. Although previous studies found girls superior to boys in number vocabulary at these ages, Martin found that boys were consistently superior when the child's spontaneous conversation was analysed for its quantitative content.

Number knowledge of beginning school children. The most complete study of children's number knowledge at this stage has been carried out by Brownell (1940). He administered a battery of tests in number ability to about 700 rural and city children at the time they were entering first grade. The tests used were similar to those employed in the studies reported previously. In addition to presenting the results of the study Brownell also reviewed several other American studies in children's number knowledge. Although the tests used differed in quality and quantity and although there were variations in the populations sampled and the representativeness of the samples, there was a

surprising degree of uniformity in the results obtained. Judging from these results, it would appear that young children have a large fund of information in the area of number before systematic instruction is begun. Brownell summarized the results of previous studies and of his own by presenting in detail the number concepts that are, first, "quite well developed by the time most children start school," second, "not so fully known to school entrants, but are fairly well started among a reasonably large per cent of children of this age," and third, "possessed by less than a third of school entrants, and then in limited degrees of richness and proficiency." (pp. 58-59)

Again the results of this study seem to substantiate those of previous studies in that girls excel boys in most of the tests administered. There seems to be agreement on the fact that city children are superior in their number knowledge to rural children. In both instances, however, the differences are small.

Very few studies have examined the relation of intelligence to number knowledge in young children. Grant (1938) analysed the results of number tests for three groups of children defined as "dull," "average," and "bright," the groupings having been made on the basis of I.Q's. from the Pinter-Cunningham Primary Mental Tests. The analysis showed a positive relationship between intelligence and the various arithmetical skills measured. Wheeler and Wheeler (1940) found a correlation of +.51 between intelligence and the ability of children entering first grade to read the numbers to 100, (presumably this refers to the reading of number symbols).

None of the studies reported attempted to determine how the child arrived at the answers he gave to the various questions on number. Only by comparing the results from one test to another can inferences be made concerning the child's understanding of the answers given.

Gunderson (1940) employed a different technique in an attempt to determine the number knowledge of a group of seven-year-old children. To determine the understanding which children had of the number two, for example, she asked such questions as these; Tell all you know about two. What is two? What does two mean? What does two make you think of? She followed the same procedure for other numbers, carefully recording the children's answers. Again, information gathered in this way would make it appear that these children had fairly extensive knowledge of number. However, no attempt was made to establish a relationship between the child's verbalizations of number and his knowledge of the underlying concepts involved.

Mascho (1960) conducted a study of the child's familiarity with measurement. His sample consisted of one hundred fifty beginning first grade children. The technique used for gathering the data was the personal interview and the instrument was, "The Test of the Pre-School Child's Familiarity With Measurement" devised by MacLatchy and mentioned previously. On the basis of this investigation Mascho concluded that:--

1. Age seemed to contribute to the child's familiarity with measurement, the younger children being less familiar with measurement than older children.

2. Socio-economic status was another contributing factor, the lower

socio-economic status children being less familiar with measurement than were the middle or upper socio-economic status children.

3. Sex and attendance or non-attendance in kindergarten did not appear to contribute to the child's familiarity with measurement.

4. There was a wide range in the child's familiarity with measurement among children of first grade level.

5. Children involved in this investigation were less familiar with measurement than those involved in previous studies reported.

It can be inferred from the foregoing studies that the child's number knowledge, as revealed by his verbalizations, is fairly extensive. However there are wide variations and many factors such as age, sex and socio-economic status which seem to contribute to the child's knowledge of number.

THE DEVELOPMENT OF THE CONCEPT OF NUMBER

Recent theories of learning have been emphasizing understanding as the basis for learning. The theory is that in order for children to understand number, the basic concepts underlying number must be developed.

This idea of the development of the basic concepts of number is not new. Judd (1927) advanced the view that the procedure of guiding the child to a complete understanding of number would be successful only when there was an intelligent analysis of the number system on the one hand and an equally intelligent consideration of the child's mode of thinking on the other. He criticized the psychological theories of

learning which were prevalent at that time, claiming that they provided no explanation of the generalized or abstract idea of number and that they represented the learning of number as the acquisition of a long list of combinations, sometimes called "number facts." He further claimed that once the child is carried far enough to acquire insight into general ideas, he will be able to generalize and make specific applications on his own. When once acquired, a general idea becomes a dynamic fact in experience. The character of every mental process, Judd maintained, was determined by its organization and not by its isolated elements.

It has taken a long time for Judd's views to become accepted and acted upon, but current theories of learning have recognized the wisdom of these views, the emphasis has now been shifted from skill in computation to an understanding of the underlying concepts or as Judd terms them, "insight into general ideas."

Vinacke defined concepts as cognitive organization systems which bring pertinent past experience to bear on a present object or situation. On the acquisition of concepts he concludes that (1) increasing age is the most important factor (2) progress is continuous rather than occurring in distinct phases (3) earlier learning provides for later development and (4) the development of concepts proceeds from the simple to the complex, from the concrete to the abstract, from consistent to more consistent. He further claims that concept formation involves processes which cannot be inferred from either mental age or vocabulary.

Number concepts are one class of concepts: they are generaliza-

tions about the relationships between certain kinds of data. According to Lovell (1961), when dealing with number the child has to move from percepts (arising from the environment) and actions to the abstract concept. If the concept of the natural numbers does not develop in the child, if they do not exist in his mind, divorced from particular things, apparatus, actions or circumstances, the mental operations that he can perform with them will be restricted.

Children seem to develop concepts naturally, but since these must develop slowly and since they live in a complex world from which so many concepts can and should be formed, it is perhaps to be expected that their concepts should be inaccurate and incomplete.

Perhaps the most extensive investigation ever undertaken in the development of the concepts of number was that carried out by the Swiss psychologist Jean Piaget (1952). This was primarily a study of the building up in the minds of children of four to seven years old, of the idea of numbers, as distinct from the mere knack of counting and the later mere knack of school arithmetic. It attempted to link up numerical thinking with logical thinking and to show that the development of the two is closely related. His initial hypothesis was that the development of the ideas of number goes on side by side with growth in capacity for logical thought and that a pre-numerical stage corresponds to a pre-logical one, he claims that, "operations" which are necessary conditions of an understanding of number are, the ability to deal with the equivalence of cardinal classes in terms of one-to-one correspondence, and the ability to deal with transitive relations such as "less than" and "greater

than." This suggests that to understand number, the child must be able to manipulate and make judgements about perceived objects in such a way that the arrangements, size or spacial arrangement does not influence judgements about the number of objects perceived.

Piaget devised a number of ingenious experiments designed to test the child's knowledge of, and the stages of development in, such concepts as conservation of quantity, cardinal and ordinal correspondence, the properties of cardinal and ordinal number, the notion of series, the ability to classify and the relation of classes to number, the additive composition of numbers, multiplication and the co-ordination of relations of equivalence, and the additive and multiplicative composition of relations and the equalization of differences.

These experiments were tried out on what Piaget called "ordinary school children of primary school age." The method used in the experiment was what he terms the "clinical method," which was a combination of observation and interview. For example, in testing the child's knowledge of the conservation of quantity, each child was presented with two similar cylindrical vessels of coloured liquid of equal size and containing equal quantities of liquid. The contents of one of the vessels was then poured into another container which was similar in shape but taller and smaller in diameter. The child was asked whether the quantity of liquid had remained equal to that in the undisturbed vessel. The problem was presented in a variety of ways, but always in terms of the preservation, or not, of equality with the liquid in the standard vessel. The results showed that at first the idea of a constant quantity in changing forms was not present but that it was gradually built

up. Three stages of development could be distinguished.

In the first stage, at about four to five years of age, the child considered it natural that the quantity of liquid should vary with the shape of the vessel into which it was poured. The child had no idea of an invariable quantity which would correct the visual impression at this stage. The second stage, usually found in children between five and a half and six years of age, was one of transition and elaboration. The idea of conservation could be seen to be dawning, but though it was discovered in the course of certain pouring operations, it was not generalized and it broke down in certain circumstances. There appeared to be a conflict between the child's knowledge and his perceptions, with the latter being the dominant factor. In the third stage, at about six and a half to eight years of age, the concept of conservation was firmly established, the child assumed from the start that the quantities of liquid were constant. According to Piaget, at the first stage the child's concept of quantity is determined by his perceptions. If the level of the liquid in one vessel is higher than in the other, the child believes the quantity must be greater. He cannot take two dimensional relationships into account and reason that, although the level is higher, the total quantity is preserved since the diameter of the vessel is smaller. At the second stage, the child is arriving at a logical notion of quantity, but he is still rather uncertain and if the liquid is poured into two or more containers, he loses the idea of conservation and believes that the quantity has changed. When the third stage has been reached the child is ready to understand total stable quantity independent of

appearances or arrangement. This last discovery alone makes possible the true development of number.

In the other experiments on number concepts, Piaget found the same three stages of development. They may be summarized as follows:--

1. The first stage is characterized by what Piaget calls "preoperational thought" and is evidenced by children of four to five years of age. The fundamental concepts of quantity, series, ordinal and cardinal number are not present.

2. The second stage Piaget terms the stage of "intuitive thought." This is a transitional stage and is found in children of five and a half to six years of age. The fundamental concepts of number are beginning to dawn, but the child cannot generalize. Logical reasoning cannot be employed.

3. The third stage is the stage of "concrete operational thought." This stage is reached by children of six and a half to eight years of age. Children at this stage are capable of reasoning logically in the field of "concrete operations." The fundamental concepts of number have become firmly established.

A great deal of interest has been shown in Piaget's work and some criticism of it has also been forthcoming. The criticism hinges mostly on the methods employed in his studies and on the analysis of his data. His work is written in an abstract and often highly technical vocabulary which he does little to explain. He draws freely on the language of formal logic and seems to assume that his readers will be familiar with his own theoretical thought. He fails to provide information about the

way in which his data were collected, or even about the number of children tested, nor does he employ rigorous statistical procedures in his analysis. He reports his findings in terms of three stages but these are not necessarily three stages in the growth of the same children. In some instances the same child does turn up at a later stage but by and large his subjects are different for each experiment and also for each stage.

Despite its limitations, however, the work stands out because it suggests a clear picture of the psychological processes involved at various levels of intellectual behavior, and at the same time offers an account of the way in which these are elaborated in the course of maturation and experience.

Some attempts have been made to verify the findings of Piaget's work, at least in part. Holmes (1963) carried out a study to investigate the nature of young children's concepts of cardinal and ordinal number. She was interested in (1) the developmental level of cardinal and ordinal number concepts of pre-first grade children (2) the change in the developmental level of number concepts as children grow older, and (3) the association between cardinal number concepts and ordinal number concepts. Her sample consisted of 144 children between the ages of three and six years. Her test was based on procedures used by Piaget. She concluded from this study that pre-school children lack an operational concept of number. Nursery school and pre-nursery school children showed very little knowledge of number concepts. At the kindergarten and pre-school levels the development of the number concepts

is characterized by incompleteness and inaccuracy.

Dodwell (1960) conducted a study to assess the generality of the types of behavior described by Piaget for children between the ages of five and eight years. He sought to examine age trends, the consistency of behavior at any particular age and to assess these factors as evidence for a theory of cognitive development in terms of the three stages described by Piaget. His sample consisted of 250 children who were attending school and was distributed over the grades, kindergarten to grade three. His tests were based on those used by Piaget. The interview technique was used and the children responses were scored and further classified into three groups, A, B, and C. An A response was interpreted as one which clearly indicated that the child was basing its judgement on perceived characteristics. Category C indicated that the child was in complete possession of the concept involved. While those responses which indicated that the child was at neither of those extremes were classified as a B. Dodwell found that the type of responses described by Piaget was prevalent in his study. However he discovered that a child may be in stage A for one test situation and in stage B or even C for another. His findings confirm Piaget's contention that young children do not fully understand the concept of number, even though they may be able to count.

Estes (1956) conducted a study using tests similar to those used by Piaget. The findings of the study appear to refute those of Piaget. No stages were found in the development of number concepts. According to Estes twelve of the fourteen four-year-olds and eighteen of the twenty

five-year-olds had the correct concept of number. However there may be a difference in what Estes accepts as the concept of number and the interpretation placed on it by Piaget. For example in one of the test situations used by Estes, the children were asked to count ten chips placed (a) in a row, (b) in a pattern and (c) in a loose pile. His conclusion was that, either children could count all of these or none of them. But does this really mean that the child has or has not the concept of number as Piaget interprets it? According to Isaacs (1960) the most startling thing which Piaget's studies on number demonstrate is the great gulf for the young child between being able to count and even the most rudimentary real idea of number.

Wohlwill (1960) conducted a study to determine the sequence through which an individual passes in the course of the development of the number concept. The sample consisted of seventy-two children, ages four to seven years. The technique used was Scalogram Analysis, in which a series of tasks were used which demanded behavior thought to be acquired in a sequential pattern. He then analysed the patterns of successes and failures on these tasks by individuals at varying developmental levels. The three stages in the development of the concept of number were supported by this study.

Lunzer (1962) reports a marked awakening of interest in England in Piaget's work, and in particular in the educational implications of his findings. Much of this interest has taken the form of a repetition of his studies in number and in general it can be said that his findings have been verified.

Churchill (1958) devised a number of tests similar to those employed by Piaget. She was concerned not so much with the verification of Piaget's theoretical formulations as with their applicability to educational practice. Two groups, each consisting of eight children aged five, were matched for their understanding of number concepts in the light of their performance on several test situations derived from Piaget. One of these groups was given a special program of number experiences over a period of four weeks. These sessions, which occupied two half-hour periods each week, were devoted to guided play situations in which the children were given the opportunity to experiment with a suitable assortment of toys and shapes. They were encouraged to discover for themselves the invariance of numerical relations. At the end of the period both groups were given another battery of tests. The experimental group was found to have made a highly significant improvement as compared with the control group, as judged by the number of questions which were answered at an "operational" level. The sequence of stages of development described by Piaget was borne out by this study, and the majority of the five-year-olds showed the same lack of conservation as Piaget's younger subjects. The experiment strongly suggests that education and environment can do much to accelerate the progress of children in their understanding of number.

Williams (1958) sought to test children's understanding of conservation, along with their mastery of the cardinal and ordinal properties of number. He devised a number of tests similar to those used by Piaget. The tests were administered to a group of educationally subnormal children and also to a group of pre-school children. He noted

that children who had reached a deceptively high standard in their verbalizations of the number names often showed a lack of understanding of the nature of number relationships. He concluded that under such circumstances continued formal work in number operations might be positively harmful. A similar study by Slater (1958) again showed that children of five years of age were in the main unable to perform the simplest tests demanding a real understanding of number.

Hyde (1959) repeated a large number of Piaget's experiments in number with a multi-racial group of children. The subjects were forty-eight European children, forty-eight Arabs, twenty-four Indian and twenty-four Somalis. Each group was made up of equal numbers of children at ages six, seven, and eight. Again, generally speaking, the results confirm Piaget's findings. Hyde notes that not only was there growth with age, in the direction of operational reasoning, but the answers given in Arabic were often verbatim equivalents of answers given in French by Genevan children at comparable levels.

There appears to be a fair amount of evidence to support three stages in the development of the concept of number. However, there is conflicting evidence as to whether a child at one stage in some particular aspect of number is also at the same stage in some other aspect.

III. DISCUSSION AND SUMMARY

It can be inferred from the foregoing that much attention has been given to research in the young child's knowledge of number. For the most part, attempts to determine the child's number knowledge have

been based on his verbalizations of number names and combinations. Piaget's work has pioneered an attempt to determine the relationship between the child's verbalizations and his true knowledge of the underlying concepts of number. It would appear that there is a wide gap between what the child says he knows and how much he knows about what he says.

From an overall survey of the research reported, the following inferences may be drawn:--

1. Concepts of number develop slowly out of percepts, memories, images and through the action and interaction of the child with his environment. Since each concept involves differentiation from other unlike ideas and appreciation of common relationships in members of the same class, the factor of insight plays a large part in concept development.
2. Children develop a large number of concepts even in their pre-school years, not all of them however are complete and accurate. Both in-school and out-of-school experiences continue to operate in increasing understanding of environmental objects, qualities and relationships.
3. There appears to be a definite serial order in the development of the concept of number.
4. Although the growth of number concepts is orderly, any group of children shows a wide range in their understanding of the concepts.
5. Knowledge of number concepts is related closely to background of experience and is positively related to such factors as chronological age, mental age, socio-economic level and in some studies sex has also been shown to be a related factor.

6. Since facility with the number names exceeds the development of the underlying concepts of number, it can be a very deceptive predictor to use in determining the child's knowledge of number. Children may know a concept thoroughly, partially, inaccurately, or not at all.

CHAPTER III

DESIGN OF THE EXPERIMENT

This chapter will be devoted to a discussion of the procedures followed in conducting the investigation and the statistical procedures employed in the analysis of the data.

I. BACKGROUND OF THE STUDY

This investigation was carried out in the city of Edmonton, Alberta, Canada, where the City of Edmonton Recreation Department, in co-operation with a sponsoring organization such as a Community League, or an interested group of parents, operates a number of playschools. The primary purpose of these playschools is to aid in the development of the pre-school child, socially, physically, emotionally and intellectually. The program consists of the following five phases:--

1. Art and Handicrafts
2. Physical Activities
3. Storytelling and Drama
4. Music
5. Special Events

Playschool classes are conducted for three, two-hour sessions each week for a period of seven months each year, October 1st to April 30th. To be eligible for enrollment children must have reached their fifth birthday on or before December 31st of the year in which they enroll, thus making them eligible for enrollment in regular school classes the following year. A small monthly fee is imposed, and in addition, parents

of all children are expected to undertake certain responsibilities in connection with the operation of the schools.

Permission was obtained from the director, Mrs. Seeger, to conduct the study in these schools. A random sample of playschool classes was selected and a series of tests was devised and administered in these schools by the investigator during the months of March and April, 1963. The tests were then scored and the scores were punched on IBM cards for processing by the IBM computer services, University of Alberta. A detailed description of the procedures followed is presented below.

II. POPULATION AND SAMPLE

The population consisted of all children who were attending playschools in the city of Edmonton during the year 1962-63. The records of the Director of Playschools revealed that there were one hundred twenty-three classes representing approximately three thousand children registered as of February, 1963. All of these children will be eligible for enrollment in first grade in regular schools in September, 1963. According to the Directors of Elementary Education for both Public and Separate Schools in Edmonton, the total number of students expected to enroll in first grade in September 1963 is approximately 6,000. Thus the population sampled represented about 50% of all prospective first grade children in Edmonton for the 1963-64 school term.

The sample was selected by assigning a number to each of the one hundred twenty-three classes of playschool children, and selecting seven numbers representing seven classes, from the table of random numbers

given by Dixon and Massey (1951, p. 291). The total number of children thus selected was one hundred twenty-eight, but of this number two had to be excluded because they refused to submit to all of the tests, while two others were absent from school during the testing program due to illness. Thus the sample consisted of seven randomly selected classrooms of playschool children representing a total of one hundred twenty-four children.

It might be argued that playschool children represent a select group, since there is a small fee for attending these classes, and the parents of the children are expected to participate to some extent. However, according to the Director of these schools, neither of these two factors acts as a deterrent to parents who want to send their children to these schools.

From the investigator's own observations of the seven areas in which the tests were administered, it would appear that the seven classes involved represented three socio-economic levels. Two classes were from areas considered to be high socio-economic, three were from areas considered to be low socio-economic, while the remaining two classes were from areas between those two extremes.

III. THE INSTRUMENT

Since the test was designed to measure the underlying concepts of number, rather than the child's facility with number names, a review of tests used in previous studies failed to be much help, in its preparation. However, some of the experiments devised by Piaget and mentioned

previously were considered suitable for purposes of this investigation, and items similar to those used by him were constructed, together with other items which the writer felt would be suitable. A rough draft of the test was thus devised.

In order to refine the test and to phrase the questions in language which would be comprehensible to the subjects to be used in the investigation, it was administered to a group of twelve students who were attending playschool and who did not form part of the sample. On the basis of this study, the test was further revised, and the difficulties revealed by this preliminary investigation were rectified. Many of the questions were restated, and some questions which were considered to be overlapping were omitted.

As a result of this preliminary investigation, the final draft of the test was comprised of six subtests, each designed to determine the child's knowledge of a particular concept. The number of items in each subtest ranged from four to seventeen. A copy of the test is given in Appendix A. The following is a description of the various subtests, together with names of the concepts they were designed to measure:--

I. Rational Counting

This subtest was designed primarily to determine the child's knowledge of the number names, and whether or not he could make a one-to-one correspondence of the number names with the objects counted. It consisted of four items, (1) counting by one's (2) counting by two's (3) counting by five's and (4) counting by ten's. The objects used were coloured discs and sticks tied in bundles of two's, five's, and ten's.

II. Comparisons

This subtest was designed to test the child's ability to compare equal and unequal groups, by making a one-to-one correspondence of the objects in one set with those in another. It is actually a more immature process than rational counting, since it enables the child to talk about the equality or the inequality of groups without actually using the number names. For example, the child was presented with a group of eight horses and eight riders. He was then asked to determine whether there was a horse for each rider. The child was encouraged to manipulate the objects in any way he chose, to find the answer.

III. Conservation of number

This subtest was designed to determine whether children had grasped the idea of the invariance of given number. For example, the child was presented with two equal groups of red and blue blocks (15 of each). The two groups were placed close together and in one-to-one correspondence with each other. Every attempt was made to ensure that the child saw the equality of the groups. When the experimenter had satisfied himself that this had been achieved, the blue blocks were spaced about one block apart. The child was asked whether there were still as many blue blocks as red blocks. Similar experiments were carried out with groups of dolls and rattles which were unequal in size. Conservation of quantity was also studied using liquids. Although the use of liquids really involves a knowledge of measures, a concept with which this study did not concern itself, items involving the use of liquids were included in a further attempt to determine whether students

were confusing quantity with some physical attributes of the objects involved.

IV. The cardinal property of number

This subtest was designed to determine whether the child could recognize a group of objects consisting of a number named by the experimenter. This subtest was actually made up of two parts. In the first part the child was presented with eight cards of coloured buttons ranging in number from two to nine and arranged on the cards in domino pattern. The child was asked, for example, to point out a card with five buttons on it, three buttons etc. For the second part of this subtest, small blocks were used to make up the groups from two to nine. The blocks were arranged in random pattern within each group and the groups themselves were also randomly arranged. Again the child was required to point out a group of five blocks, three blocks etc. A chart of the arrangement of the groups and of the arrangement of the objects within each group was made and followed for each class of students tested to ensure that the arrangement would be the same for all.

V. The ordinal property of number

As the name suggests, this subtest was designed to test the child's knowledge of ordinal number. The objects used for this subtest consisted of ten toy horses arranged in single file and supposedly running a race, a toy fireman and fireman's ladder, a simulated parking lot on which fifty toy cars were parked in five rows of ten cars each. The child was required to do such things as finding the horse that was in third place, having the fireman climb up the ladder to the third step,

finding the third car in the second row and so forth.

VI. Place value in the decimal system of notation

It is generally recognized that young children can count, at least they know the number names in sequence up to a point. Do they have the concept of how the numbers are built up in the decimal system of notation? This subtest was designed to find out whether they have grasped this idea. For example, using bundles of ten sticks and several single sticks, the child was asked to make up groups of 11, 19, and 25.

IV. THE TESTING PROGRAM

The testing program was undertaken in the playschools during the period March 15th to April 30th, 1963. The tests were administered to individual students by the writer, with the help of an assistant, who recorded the child's answers and otherwise helped out with the setting up of the testing stations and rearranging the objects after each test. The tests were given at each of seven separate schools and in each case the testing program was carried out in a secluded room apart from the classroon so that there would be no interruptions. Only the child being tested, the writer, and his assistant were present.

Six testing stations, corresponding to the six subtests, were set up. The objects to be used for each of the tests were arranged at six separate tables or where this was not feasible they were arranged at six stations on a large table. Careful attention was paid to seeing that the test situation did not vary from one class to another. In each case the writer talked with the group before inviting any of them to submit

to the test. Having described the testing situation to them, all children were most enthusiastic about trying it. When the child entered the testing room he was invited to have a look at all the toys displayed on the tables. This period which usually lasted a minute or two was designed to put the child at ease. When this was done the testing began, beginning with the first subtest and continuing in sequence through all of the subtests. The actual test required from fifteen to twenty minutes and in all cases the child's interest was maintained throughout. A separate form (see Appendix A) seeking information on the child's date of birth and other relevant information was left with the class teacher, to be completed by the parent and returned to the writer.

V. THE ANALYSIS TO BE USED

In the first subtest which involved object counting, some difficulty was experienced in trying to arrive at some method of scoring. Several methods used by other writers were examined, none of which were acceptable to the writer. In collaboration with a specialist in mathematic's education and a research specialist, it was decided that these items would be scored by assigning the logarithmic (base 10) value to each counting score and multiplying by ten. The rationale behind this method was that it required more ability to count to ten than to proceed in decades beyond that.

The remaining items (items 5 to 53) were scored by assigning a one or a zero to each one, depending upon whether the child was able to arrive at the correct answer or not. The assumption was that if the

child was able to manipulate the objects involved and arrive at the correct answer, then he had some knowledge of the concept involved and he was therefore assigned a score of one for that particular item. If he could not arrive at the correct answer, the assumption was that his knowledge of that particular concept was inaccurate or inadequate and a score of zero was given. These data were then punched on IBM cards and the analysis described below was carried out by means of the IBM 1620 computer.

Since the subtests were set up arbitrarily by the writer, it was decided that some form of statistical analysis should be employed to determine whether they were independent of each other or whether there was overlap between them. Because the first subtest was designed to test the child's verbalizations of the number names, it was considered to be independent of the skills to be tested in the remaining five subtests. It was decided therefore to find the intercorrelations (phi-coefficients) of all the remaining items and to factor-analyse these phi-coefficients in an attempt to determine the degree to which these subtests were independent.

According to Fruchter (1954), factor analysis is a method of analysing a set of observations from their intercorrelations to determine whether the variations represented can be accounted for adequately by a number of basic categories smaller than that with which the investigation started. If the subtests as arbitrarily set up, were indeed independent, then it would be expected that the factors resulting from this analysis would correspond fairly closely to the subtests. Only

those items having high factor loadings in a particular subtest would be considered in arriving at a final score for that particular test. Those items having low loadings would be discarded. The student's scores for each factor were then adjusted by considering in the total scores for any factor only those items which had high loadings on the particular factor in question.

The individuals comprising the sample were reclassified into various groups on the following bases:--

1. Sex. Students were classified according to sex at each of three age levels.

   (a) Below five and a half years.

   (b) Above five and a half and below six years.

   (c) Six years and over.

2. Socio-economic status. Students were classified into two groups, high socio-economic status and low socio-economic status. This was determined on the basis of the father's occupation. All children whose fathers were in professional, managerial, or clerical positions were classified as high socio-economic status, while those whose fathers were unemployed, labourers, mechanics, equipment operators, or garage workers were classified as low socio-economic status. Generally speaking all white-collared workers were regarded as being of high socio-economic status, while all others were placed in the alternate category.

3. Children with brothers and/or sisters attending school and those with no brothers and/or sisters attending school.

4. Age. The total sample was divided into three age groups as follows:--

(a) Below five and a half years.

(b) Above five and a half and below six years.

(c) Six years and over.

To determine the relationship between the child's verbalizations of number and the knowledge of number concepts, the intercorrelations of counting scores with factor scores will be calculated.

An attempt will be made to determine which of the factors enables the distinctions between the groups, described previously, to be most clearly made. Since several variables have been measured on each individual, it may be found that some linear function of these variables is more effective in distinguishing the groups than is any one of them. To carry out this statistical procedure, the discriminant function will be used. This particular statistic was developed by Fisher (1936). According to Johnson (1949), "the essential property of this function, which is a linear function of the observations, is that it will distinguish better than any other linear function between the specified groups on whom common measurements are available."[2] It is estimated from the data as that function for which the ratio of the sum of squares between groups to the residual sum of squares is a maximum.

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[2]Palmer I. Johnson, Statistical Methods in Research, Prentice-Hall, Inc., New York, 1949, p. 344.

CHAPTER IV

SUMMARY AND ANALYSIS OF RESULTS

This chapter is chiefly concerned with the analyses which were carried out. In the attempt to determine to what extent pre-school children have developed the concept of number, a sample of seven play-school classes, representing a total of one hundred twenty-four children was selected. A test designed to determine some of the basic concepts of number was devised and administered to these children individually by the investigator during the months of March and April, 1963. The tests were scored, and the results analysed in the manner to be described below.

The analysis will further seek to test the following hypotheses, at the .01 level of significance.

Hypothesis 1.

There is no significant relationship between the child's verbalizations of the number names and his knowledge of the underlying concepts of number.

Hypothesis 2.

There is no significant difference between the number knowledge of boys and girls.

Hypothesis 3.

There is no significant difference in the number knowledge of children of high socio-economic status and those of low socio-economic status.

Hypothesis 4.

There is no significant difference in the number knowledge of

children with brothers and/or sisters attending school and those with no brothers and/or sisters attending school.

Hypothesis 5.

There is no significant difference in the number knowledge of children in each of the age groups five to five and a half years, five and a half to six years and six to six and a half years.

I. ORGANIZATION OF THE ANALYSES

The analysis will be reported under the following headings:--

1. Summary of Results for Total Sample

   An overview of the results of administering a test of the basic concepts of number will be presented. No attempt will be made at this stage to differentiate between the various sub-groups represented by the sample, such as age, sex, socio-economic status and children with or without other siblings in school. This section is intended primarily to give the reader an overall view of the performance of this sample of pre-school children who will begin first grade in September, 1963. A summary of the performance of the total group on each of the various subtests will be presented.

2. Factor Analysis

   The procedure of factor analysis will be carried out to determine what factors are being measured by the test, and which of these factors are most effective in determining the pre-school child's concept of number as measured by this test.

   The subtest dealing with rational counting will not be included in this analysis because the scoring procedure used for this

subtest was different from that used for the other subtests. However the intercorrelations of counting scores with factor scores will be presented.

A brief account of the particular method of factor analysis and of rotation used will be given and the various factors thus isolated will be interpreted.

3. Discriminant Analysis

The factors isolated by the method of factor analysis will be further analysed by the use of a discriminant function analysis developed by Fisher (1936). This analysis will be used to determine the effects of age, socio-economic status, sex and siblings on the child's concept of number. It will also show which factors are most effective in discriminating between the various sub-groups.

II. SUMMARY OF RESULTS FOR TOTAL SAMPLE

Nature of the Sample

As noted earlier a test designed to measure the pre-school child's knowledge of some of the underlying concepts of number was devised and administered to a sample of one hundred twenty-four pre-school children. The sample consisted of seven classes of playschool children, randomly selected from one hundred twenty-three such classes registered in the playschools of the City of Edmonton, Alberta, Canada during the 1962-63 term.

Table I shows the distribution of the sample with regards to age and sex.

The sample consisted of about equal numbers of boys and girls whose ages varied from five years four months to six years five months.

TABLE I

AGE AND SEX OF CHILDREN INCLUDED IN SAMPLE

Age* (years & months)	Male	Female	Total
5 - 4	15	8	23
5 - 5	7	6	13
5 - 6	3	10	13
5 - 7	4	5	9
5 - 8	5	4	9
5 - 9	5	4	9
5 - 10	5	2	7
5 - 11	1	5	6
6 - 0	4	5	9
6 - 1	5	7	12
6 - 2	5	5	10
6 - 3	2	0	2
6 - 4	1	0	1
6 - 5	1	0	1
Total	63	61	124

*Age was computed to the nearest month as of April 15, 1963.

There were forty-nine children at the lower three age levels, but only four children at the upper three age levels. All four children at the upper age levels noted above were boys, while those at the lower age levels were about evenly distributed. In later analyses the age groups, five years four months to five years six months, five years seven months to five years eleven months, and six years to six years five months, are used. The numbers of children in these groups are forty-nine, forty and thirty-five respectively.

The Test

A copy of the test used in this investigation is to be found in Appendix A. It consists of six subtests each designed to measure a specific concept. The number of items varied from one subtest to another and no attempt was made to order them according to difficulty for the total test. The items in the subtests dealing with comparisons, conservation of number, and place value were ordered according to difficulty within each of these subtests. However in the subtests which dealt with cardinal number and ordinal number this procedure was not followed, since it was felt that, to do so would have introduced cues to the correct responses.

The items in subtest I which dealt with rational counting were scored by assigning logarithmic (base 10) values to the highest count attained by each child on each item. Values thus obtained were multiplied by ten and summed for the four items, giving a composite counting score. All items in the remaining subtests were scored by assigning a one or a zero to each, depending upon whether the child arrived at the

correct answer or not.

Item difficulty as expressed by the percentage of children scoring correct answers for items five to fifty-three is presented in table II. Items one to four did not lend themselves to this procedure and are therefore not included in this table.

TABLE II

DIFFICULTY OF ITEMS IN SUBTESTS II TO VI

Item Number	Per cent Successful	Item Number	Per cent Successful
5	80	30	26
6	73	31	3
7	74	32	89
8	79	33	43
9	69	34	26
10	66	35	23
11	58	36	48
12	53	37	25
13	34	38	59
14	18	39	30
15	16	40	28
16	89	41	38
17	49	42	34
18	84	43	24
19	9	44	22
20	71	45	17
21	17	46	21
22	32	47	21
23	11	48	17
24	97	49	20
25	18	50	13
26	9	51	4
27	57	52	2
28	12	53	2
29	65		

Results of Subtest I -- Rational Counting

This subtest was designed to determine the child's facility with the number names. It consisted of four items, counting by one's, counting by two's, by five's and by ten's as shown in the test in Appendix A.

The results of this subtest are presented in table III. It is interesting to note that only 3.2 per cent of the total sample were unable to count by one's to ten, while 8.1 per cent were able to count to one hundred or more. More than half of the children counted between

TABLE III

SUMMARY OF RESULTS OF SUBTEST I
RATIONAL COUNTING
(N = 124)

Counting Activity	Response	Per Cent Successful in Making Response
1. Counting by one's	Less than 10	3.2
	Between 10 and 29	57.2
	Between 30 and 49	17.2
	Between 50 and 99	14.5
	100 and over	8.1
2. Counting by two's	Unable to count	46.7
	Between 2 and 8	22.6
	Between 10 and 16	17.0
	18 and over	13.7
3. Counting by five's	Unable to count	72.6
	Counted to 10	10.5
	15 to 45 inclusive	10.5
	50 to 100	6.4
4. Counting by ten's	Unable to count	72.6
	Counted to 20	8.1
	30 to 50 inclusive	4.8
	60 to 100	14.5

ten and twenty-nine discs correctly, while another 31.7 per cent were able to count between thirty and ninety-nine correctly.

Counting by two's proved to be a little more difficult and nearly half (46.7%) of the children were unable to respond to this item. Some 22.6 per cent counted to numbers between two and eight, another 17 per cent were able to count to numbers between ten and sixteen, while 13.7 per cent counted to numbers 18 or more.

Counting by five's proved to be still more difficult for this group of children and the majority of them (72.6%) were unable to respond. Some 10.5 per cent counted to ten, another 10.5 per cent counted to numbers between fifteen and forty-five, while 6.4 per cent attained counts between fifty and one hundred.

Counting by ten's was equally difficult and again 72.6 per cent of the children were unable to cope with it at all. Some 8.1 per cent counted to twenty, 4.8 per cent counted to numbers between thirty and fifty, while 14.5 per cent were able to attain counts between sixty and one hundred.

Result of Subtest II -- Comparisons

This subtest was designed to determine the child's ability to compare equal and unequal groups of objects by making a one-to-one correspondence of the objects in one group with those in another. One-to-one correspondence is basic to the acquisition of the concept of number. The subtest consisted of six questions, and the objects used were so related as to encourage the child to match one group with another and thus to determine the equality or inequality of the two groups. (See

Appendix A for items used in this subtest.)

Table IV shows the results of this subtest. The total number of items for this subtest was six.

Well over half of the total group (58.1%) obtained scores of five or six out of a total possible score of six on this test. Only slightly more than 16 per cent had scores of two or less, while the remainder (approximately 25%) got scores of three or four.

TABLE IV

SUMMARY OF RESULTS OF SUBTEST II
COMPARISONS
(N = 124)

Score	Per Cent of Children Obtaining Scores
0	0.8
1	3.2
2	12.1
3	10.5
4	15.3
5	22.6
6	35.5

Results of Subtest III -- Conservation of Number

This test was designed to determine what effect spatial relationships have on the child's concept of number. For example, the child was

presented with a group of dolls, and an equal group of rattles, matched in one-to-one correspondence. The child was then asked the preliminary question, does each doll have a rattle? No child had any difficulty in seeing that there was a rattle for each doll. Then the rattles were moved together in a small pile, while the dolls were spread out, and the question was repeated. In all there were five of such items (see Appendix A). Two of the items, however, involved liquids, and although this would appear to be getting into measurement, with which this study was not concerned, these items were included in a further attempt to determine to what extent the child's concept of quantity is being governed by his perceptions. The results of this subtest are summarized in table V. The total number of items was five.

TABLE V

SUMMARY OF RESULTS OF SUBTEST III
CONSERVATION OF NUMBER
(N = 124)

Score	Per Cent Successful
0	36.3
1	9.7
2	19.3
3	16.1
4	8.1
5	10.5

It should be noted that some 36.3 per cent of the children showed no knowledge of the concept involved. They believed that the number of objects was dependent upon their arrangement in the group. For example, when the rattles, mentioned earlier, were moved together in a small pile while the dolls were spread out, they believed that there was not now a rattle for each doll, indicating that spatial arrangement was interfering with the concept of number. Similar confusion was noted in responses to other items in this subtest, including the two involving liquids. If the contents in one of two equal containers of liquid was poured into another container that was taller and smaller in diameter, the child believed that the quantity was greater because the level of the liquid was higher. When one five year old was queried as to why he thought there was more liquid in the taller container, his retort was, "Cause I'm five years old and I'm smart."

About one in five of the children in the sample obtained scores of four or five in this subtest which would seem to indicate that they were not confusing number with the physical arrangement of the objects involved. For example one child when asked why she thought the quantity of liquid in the taller container was the same as in the other, replied with gestures of her hands, "This one is taller, but it is skinnier, therefore it has no more in it."

Approximately 45 per cent of the children obtained scores ranging from one to three.

Results of Subtest IV -- Cardinal Number

This subtest was concerned with the recognition of the cardinal

number of objects in a group without counting. The child was required to recognize groups ranging in size from two to nine, both in pattern arrangement (buttons on a card) and in random arrangement (groups of blocks randomly arranged). There were sixteen items in this subtest. Table VI presents a summary of the results.

TABLE VI

SUMMARY OF RESULTS OF SUBTEST IV
CARDINAL NUMBER
(N = 124)

Score	Per Cent Successful
0	0.8
1	3.2
2	5.7
3	6.5
4	5.7
5	10.5
6	16.9
7	16.1
8	8.9
9	10.5
10	4.8
11	4.8
12	2.4
13	3.2
14	0
15	0
16	0

There were wide variations on this subtest, with scores ranging from zero to thirteen. Slightly more than 16 per cent of the total sample obtained scores ranging from zero to three, 68.2 per cent had scores ranging from four to nine while 15.2 per cent obtained a score of ten or more. It should be noted that the score in each case represents the number of groups identified. Further information on the size of groups recognized is given in the interpretation of factors in the next section.

Results of Subtest V -- Ordinal Number

This subtest was made up of seventeen items (see Appendix A) each attempting to seek information on the child's concept of ordinal number. Again this subtest was divided into two parts.

The first part was designed to determine the child's knowledge of ordinal number in one dimension, such as finding the fourth horse in a row of ten horses, or having a toy fireman climb a ladder to the fourth step and so forth. The second part was rather more difficult and consisted of such things as having the child find the third car in the second row in a simulated parking lot on which fifty cars were parked in five rows of ten cars each. A summary of the results appears in table VII.

The total possible score on this test was seventeen. It should be noted that slightly more than one third (33.8%) of the total students in the sample obtained a score of one or less, almost a third (32.3%) obtained scores ranging from eight to seventeen, while the others were distributed over the scores from two to seven. More information relative to this subtest will be given in the next section when the items are factor analysed and the factors interpreted.

TABLE VII

SUMMARY OF RESULTS ON SUBTEST V
ORDINAL NUMBER
(N = 124)

Score	Per Cent Successful
0	7.3
1	26.5
2	8.1
3	12.1
4	2.4
5	2.4
6	0.8
7	8.1
8	3.2
9	1.7
10	3.2
11	6.5
12	1.7
13	2.4
14	4.8
15	0.8
16	3.2
17	4.8

Results of Subtest VI -- Place Value

This test attempted to determine the child's concept of the construction of our number system. It was made up of five items and required the child to do such things as selecting a group of nineteen

coloured sticks using bundles of ten sticks and some single sticks (see Appendix A). A summary of the results of this test is presented in table VIII.

The concept of place value proved to be the most difficult for this group of children, since 79.4 per cent showed a complete lack of knowledge of the concept as measured by the test. Only 2.5 per cent obtained the full score of five, while 18.1 per cent had scores ranging from one to three.

TABLE VIII

SUMMARY OF RESULTS OF SUBTEST VI
PLACE VALUE
(N = 124)

Score	Per Cent Successful
0	79.4
1	7.5
2	8.9
3	1.7
4	0
5	2.5

Graphical Representation of Subtest Scores

In order to provide the reader with a composite picture of the performance of this group of children on each of the subtests, graphical representations of test scores are given in figures 1 to 5.

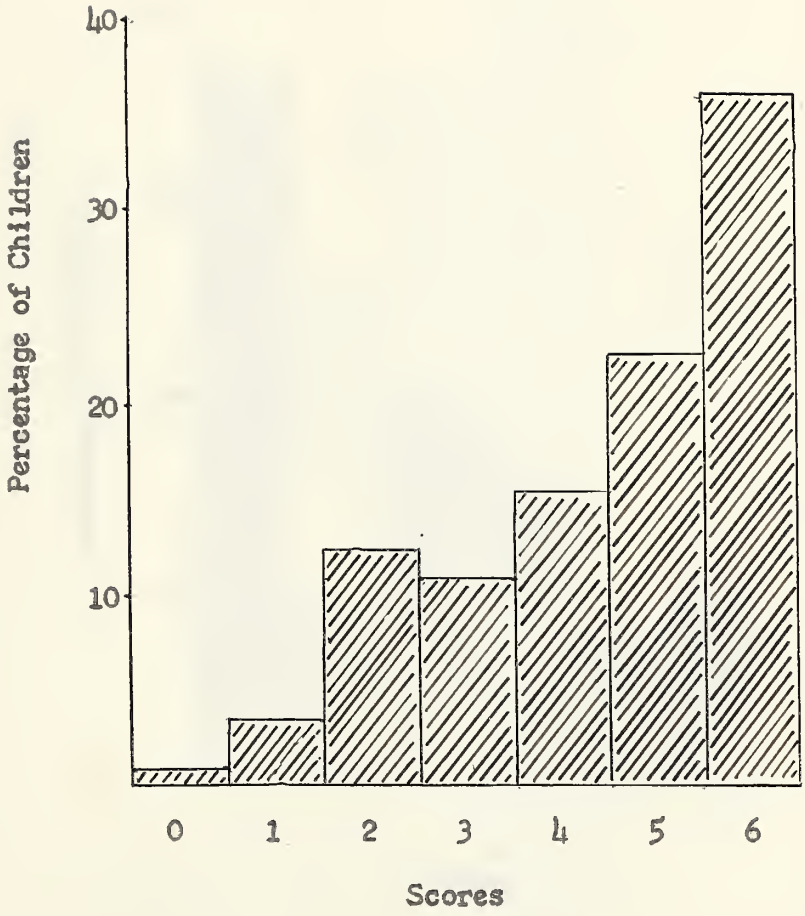

Figure 1. Graphical representation of scores. (Based on data obtained from Subtest II -- Comparisons.)

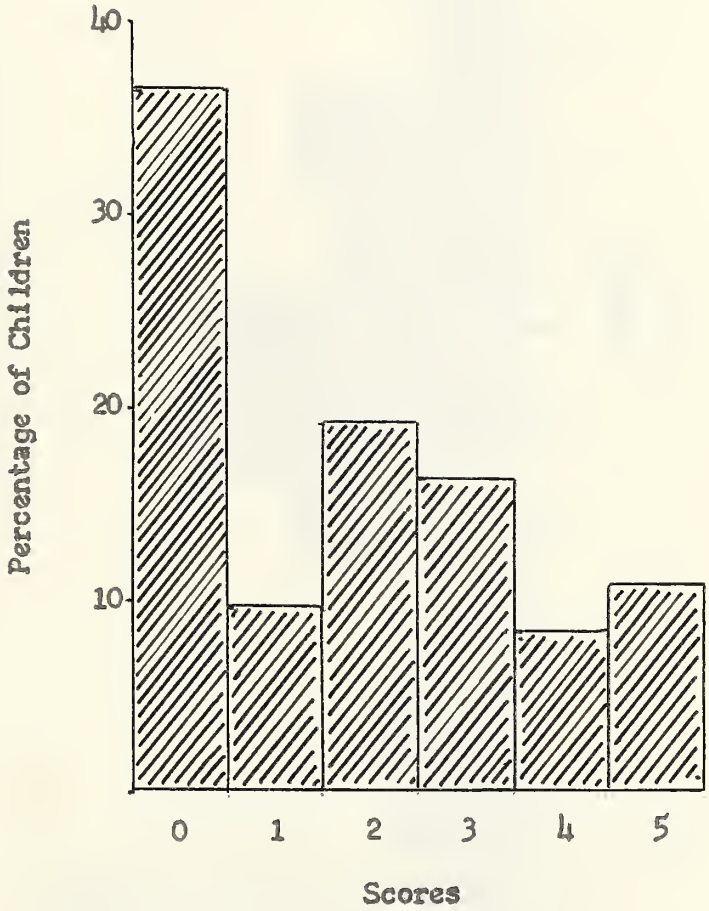

Figure 2. Graphical representation of scores. (Based on data obtained from Subtest III -- Conservation of Number.)

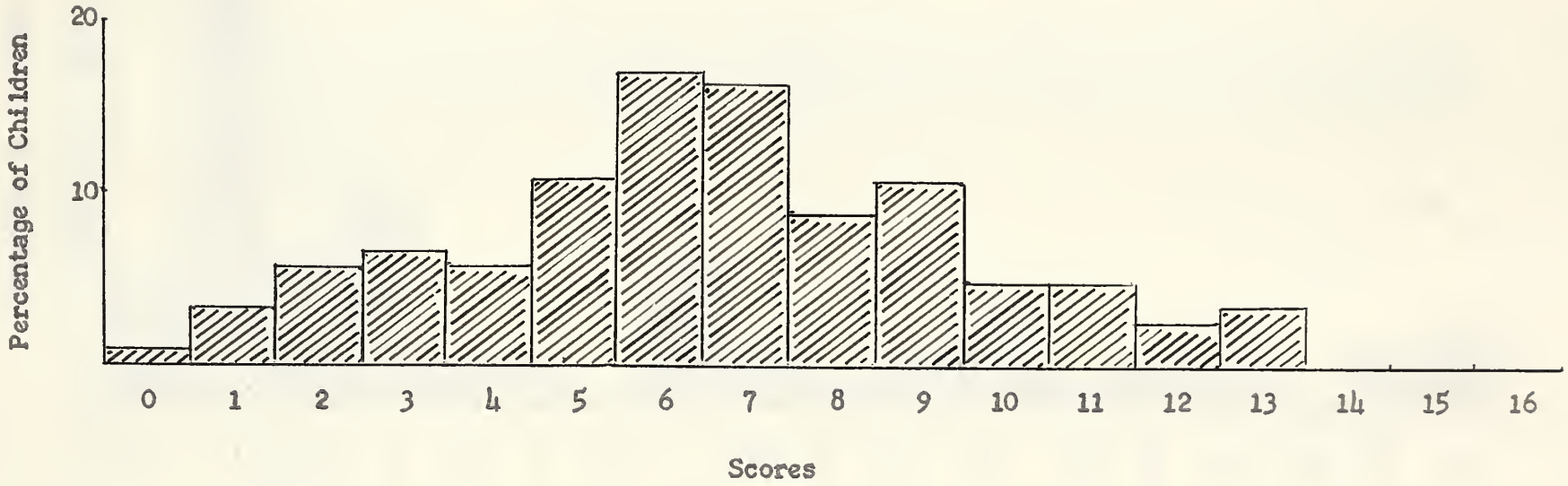

Figure 3. Graphical representation of scores. (Based on data obtained from Subtest IV — Cardinal Number

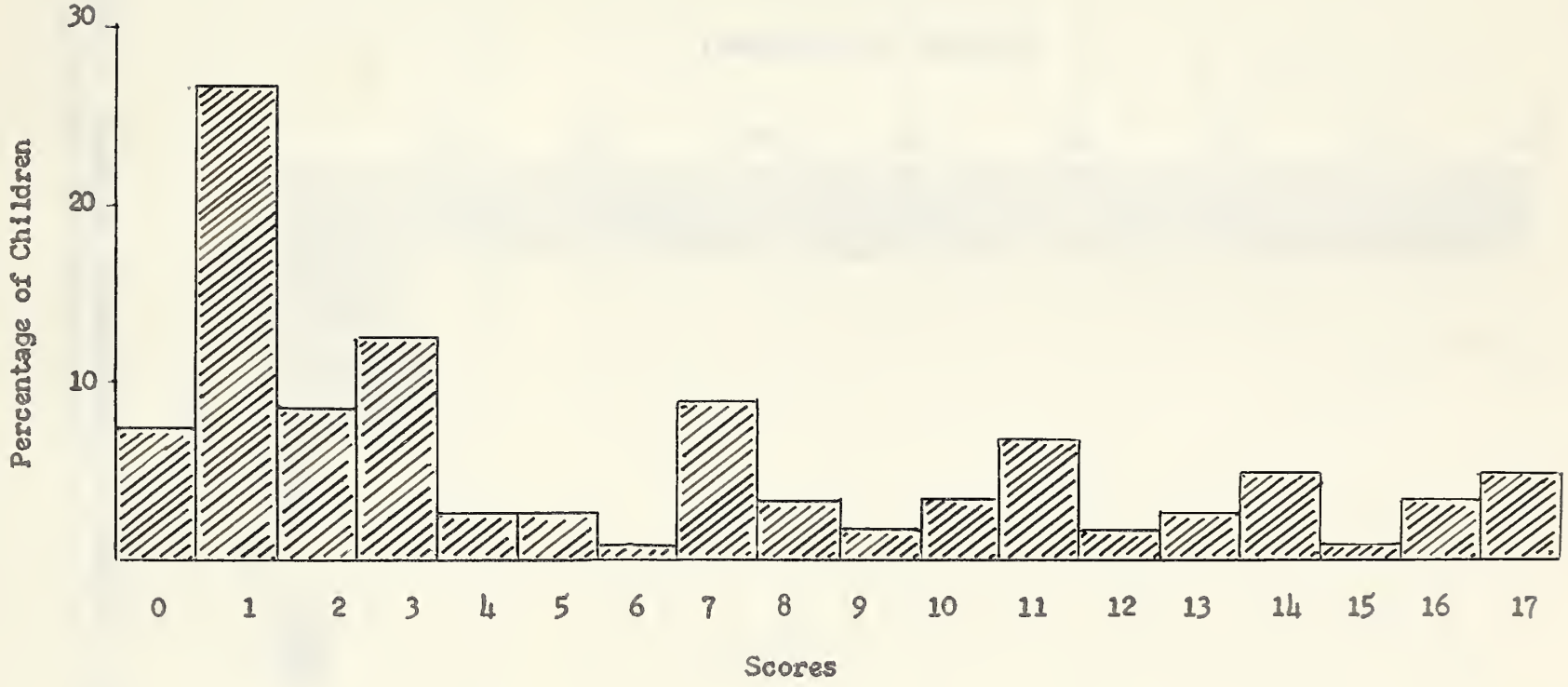

Figure 4. Graphical representation of scores. (Based on data obtained from Subtest V — Ordinal Number.)

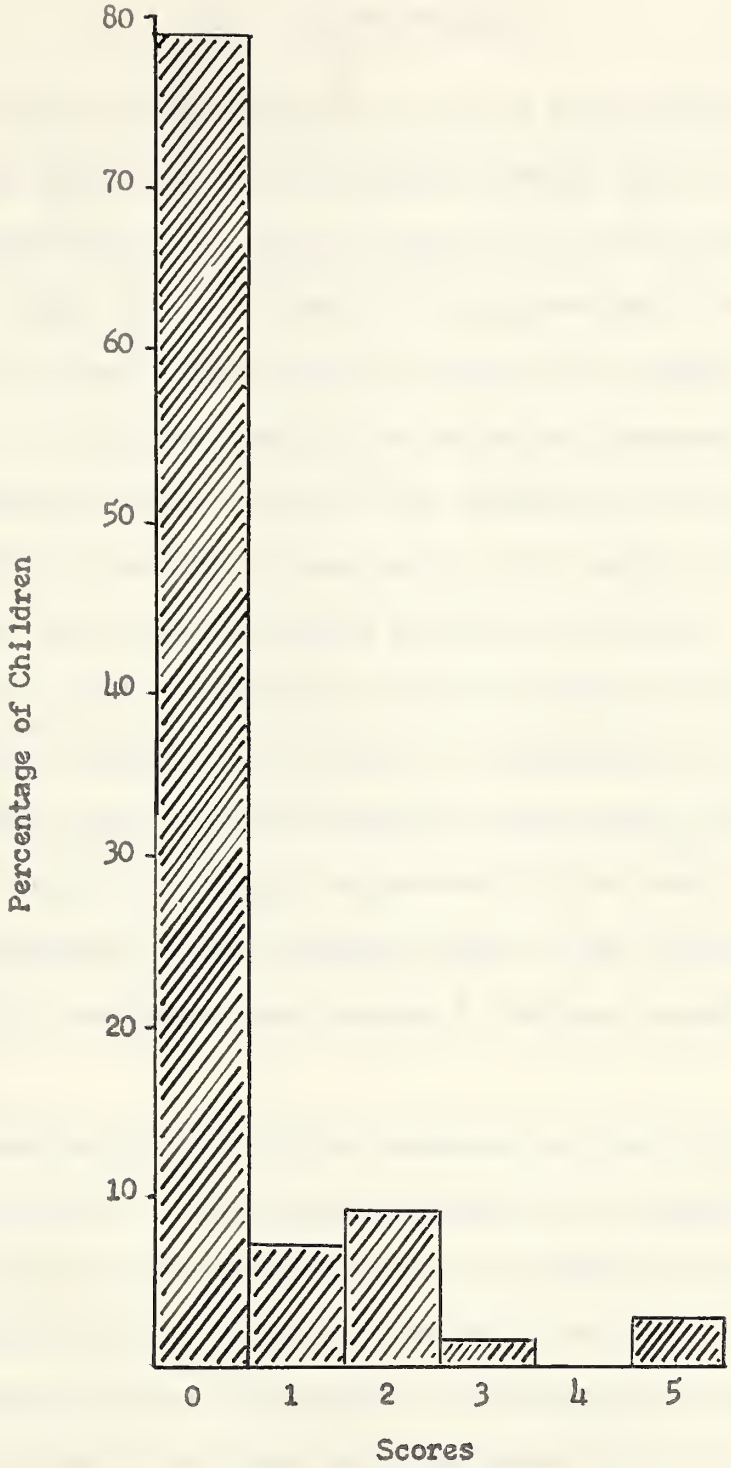

Figure 5. Graphical representation of scores. (Based on data obtained from Subtest VI -- Place Value.)

III. FACTOR ANALYSIS

The basis of factor analysis is that if two activities involve a common element there will be a correlation between them. If a number of tests are administered to a group of people, the correlation between the results will depend upon the extent to which these tests are calling upon common abilities. Thus a factor analysis of a number of variables is undertaken to determine whether the variations represented by a large number of responses can be accounted for adequately by selecting those variables that are measuring common skills. The number of categories represented by those responses would therefore be reduced. The question to be answered by this analysis was: Are the subtests as arbitrarily set up, actually measuring skills which are independent of each other, and if so, which items are discriminating between those skills most effectively? Only the variables represented by the items in subtests II to VI were considered in this analysis, since it was felt that there could be little doubt about what subtest I (Rational counting) was measuring.

The intercorrelations of the responses to items five to fifty-three were calculated. These intercorrelations are presented in table IX. It should be noted that two additional variables are included in this correlation matrix. Variable 54 refers to sex, while for these intercorrelations age was dichotomized, children under five years seven months being placed in one group and those above this age in another. Variable 55 thus refers to age. Actually all intercorrelations were phi-coefficients, since the variables for the subtests in question were

TABLE IX

INTERCORRELATIONS OF CHILDREN'S SCORES ON QUESTIONS FIVE TO FIFTY-THREE TOGETHER WITH THE VARIABLES SEX AND AGE*

Test Items

	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
5	1.000	167	213	008	117	090	055	113	143	111	132	032	082	010	-012	101	031
6		1.000	418	098	193	236	175	167	094	087	090	045	055	079	-019	028	102
7			1.000	061	352	236	212	167	170	087	016	045	237	003	043	312	102
8				1.000	320	288	202	294	218	119	138	079	-020	018	005	134	039
9					1.000	574	202	148	117	170	125	126	035	057	-010	166	090
10						1.000	121	200	008	084	075	211	017	082	027	207	203
11							1.000	760	480	340	293	104	164	-030	130	215	218
12								1.000	564	439	345	-033	162	-039	116	274	208
13									1.000	522	395	-011	153	098	163	269	213
14										1.000	645	032	122	010	081	194	105
15											1.000	018	000	034	116	105	-006
16												1.000	137	239	-017	240	022
17													1.000	358	136	412	258
18														1.000	101	430	162
19															1.000	175	361
20																1.000	164
21																	1.000

	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
5	-003	014	-011	094	027	204	-147	158	-010	127	027	176	018	102	130	062	171
6	035	-131	-035	-055	133	080	-131	021	-068	-008	197	041	074	139	137	085	127
7	191	071	-035	041	027	153	-073	060	-068	077	037	078	032	139	028	042	127
8	097	020	-013	049	051	062	020	014	114	027	051	-013	-061	-102	-173	-139	-052
9	095	-012	-048	103	-008	073	099	104	145	-028	100	256	163	181	116	130	226
10	130	007	-055	125	069	132	115	064	136	-039	082	191	074	105	063	-003	109
11	333	163	076	124	170	234	059	092	187	-006	033	079	160	066	046	119	081
12	315	156	095	182	037	252	080	132	300	023	008	124	199	136	080	190	070
13	352	115	-051	150	160	173	019	032	183	-033	019	090	136	038	039	010	058
14	263	014	-011	040	-027	127	-014	014	282	-032	027	072	121	067	072	105	078
15	158	-017	002	-003	-021	151	053	089	115	048	021	153	232	069	168	213	099
16	080	-007	120	025	-023	169	-007	203	064	-065	023	019	058	047	049	050	079
17	360	229	105	211	057	294	102	322	340	-037	028	212	274	210	210	264	213
18	132	116	297	168	015	296	-025	349	142	061	064	210	180	156	231	214	236
19	321	041	-037	384	042	085	099	038	158	-010	-042	-007	112	020	049	038	024
20	346	195	076	268	101	556	139	410	214	016	213	216	175	156	243	145	204
21	368	450	001	297	124	079	077	058	242	049	027	045	069	106	014	096	-059
22	1.000	426	-055	392	173	319	074	224	321	121	129	155	081	219	143	083	100
23		1.000	-027	301	113	122	155	073	286	098	066	052	013	013	045	007	039
24			1.000	004	-043	132	-027	161	-040	-087	228	083	035	025	108	028	135
25				1.000	226	202	001	183	225	141	034	154	055	142	119	033	131
26					1.000	064	066	019	-025	-004	047	012	027	072	062	077	-085
27						1.000	-006	548	206	153	166	140	068	199	275	098	220
28							1.000	115	371	098	-023	052	073	014	-056	007	-013
29								1.000	118	125	160	253	170	202	368	234	355
30									1.000	067	025	146	109	119	044	106	051
31										1.000	004	071	-008	008	034	092	002
32											1.000	184	156	139	232	077	287
33												1.000	604	581	687	598	573
34													1.000	486	501	682	484
35														1.000	524	488	404
36															1.000	537	686
37																1.000	338
38																	1.000

	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	Sex 54	Age 55
5	104	095	088	084	118	085	085	077	085	187	085	036	-049	-011	-011	-075	033
6	004	043	057	074	137	119	102	108	163	090	008	-013	077	035	035	-033	-082
7	084	084	094	074	158	119	102	153	163	090	008	-013	077	035	035	004	-155
8	-015	-052	020	011	058	022	-014	012	-027	029	022	-016	152	248	248	069	-052
9	100	134	230	281	101	160	137	149	160	125	160	178	092	048	048	170	-024
10	170	125	160	166	129	163	203	151	163	145	184	143	101	055	055	086	-006
11	118	063	168	123	171	195	152	181	194	137	155	071	-040	031	031	344	-019
12	202	141	233	212	243	260	223	245	260	213	260	144	-013	010	010	254	178
13	067	198	164	172	205	108	168	207	190	233	231	124	020	060	060	048	005
14	117	202	206	176	282	187	214	300	286	285	187	177	049	011	011	-007	-007
15	137	222	199	205	203	141	153	205	246	285	141	224	-034	-002	-002	090	081
16	072	061	096	080	053	041	022	038	041	018	041	002	-078	-119	-119	028	049
17	385	325	316	257	279	176	194	238	215	132	254	144	000	105	005	130	-048
18	259	236	263	228	220	143	162	192	197	156	143	064	-030	-006	-006	064	-060
19	197	166	138	118	032	158	216	166	158	042	224	077	141	215	215	050	-049
20	281	245	305	250	217	135	164	169	222	153	266	215	083	040	040	007	-115
21	260	156	135	140	137	231	197	190	179	211	283	211	071	139	139	107	014
22	276	221	264	198	148	169	185	185	127	159	252	190	074	057	057	087	074
23	150	066	142	128	000	-028	009	035	028	017	121	053	121	193	193	093	-006
24	048	038	069	055	030	019	001	017	019	-002	-019	018	101	146	146	-015	003
25	241	140	290	212	073	267	297	176	215	256	420	451	281	270	270	040	-007
26	008	-031	-019	-008	-016	-007	-027	-013	-007	-021	-007	078	-008	-043	-043	-058	052
27	210	156	226	180	226	211	166	197	211	173	171	108	-034	-026	-026	017	-052
28	039	-019	036	007	000	028	077	129	121	-016	121	129	-008	027	027	068	045
29	301	258	290	296	186	200	171	167	200	158	179	104	-020	-051	-051	054	029
30	205	097	175	054	052	060	051	073	060	-031	320	240	099	076	076	065	-064
31	054	-012	095	041	000	109	049	-022	018	-047	109	193	049	087	087	016	041
32	177	160	215	189	147	062	103	126	130	098	062	078	-008	-043	043	087	062
33	559	584	669	658	329	295	327	316	334	308	334	324	030	129	129	227	002
34	629	734	639	682	368	301	312	317	390	282	346	313	016	083	083	327	-029
35	459	515	569	584	341	355	411	371	401	379	309	299	129	099	099	195	-104
36	532	478	686	606	400	421	380	408	461	410	343	307	079	102	102	242	-032
37	482	582	529	564	370	341	297	310	341	264	341	288	028	095	095	248	-159
38	431	459	602	570	426	342	356	410	421	344	262	199	046	078	078	262	095
39	1.000	748	688	648	465	455	446	431	455	422	412	396	175	180	180	287	-046
40		1.000	694	779	480	377	445	438	464	419	377	357	196	197	197	195	-009
41			1.000	811	566	534	512	557	615	494	393	369	136	147	147	275	-009
42				1.000	544	543	528	522	605	554	335	318	161	168	168	190	-028
43					1.000	643	658	824	756	684	327	222	024	-030	-030	075	-028
44						1.000	778	760	787	752	384	380	140	108	108	086	-067
45							1.000	797	830	825	439	404	181	139	139	107	-071
46								1.000	808	771	400	334	146	112	112	069	-008
47									1.000	805	432	380	140	108	108	086	-067
48										1.000	459	420	077	002	002	045	-051
49											1.000	701	339	235	235	086	050
50												1.000	472	331	331	132	042
51													1.000	635	635	-020	-003
52														1.000	829	-015	-003
53															1.000	-015	-003
54 (Sex)																1.000	-018
55 (Age)																	1.000

*For variable fifty-five, age, students were classified in two age groups:

(a) below five years eight months, and
(b) above five years eight months.

treated as dichotomous and were scored one or zero. Thus a response positions the individual in one of two possible groups.

The absence of large intercorrelations among the variables indicates that the items are to some degree independent, however the presence of lesser ones prompts a search for more basic dimensions in terms of which the students respond.

The Principal Components Method of Analysis

The principal components method of analysis, that was used in this study, is such that the factors are represented by the principal axes of ellipsoids formed through the plotting of scores on the individual variables. The first principal factor accounts for the maximum possible variance, each successive one accounting for a maximum of the then existing residual variance. A unique mathematical solution for a given correlational matrix is thus produced and the initial correlation matrix reduced to the smallest number of orthogonal factors. The variables are described in terms of uncorrelated common factors.

In the course of this analysis, a number of solutions to an nth degree polynomial equation are sought, each designated by an eigenvalue for which there is a corresponding eigenvector. The factors of the original correlation matrices are represented by the product of each eigenvector and the square root of the corresponding eigenvalue, the number of factors obtained being as a consequence equal to the number of variables. Since a relatively small number of factors taken together account for the greater portion of the variance, only these are interpreted.

The unrotated factor loadings for each item on each factor having an eigenvalue greater than one were thus calculated and are shown in table X.

Varimax Rotation

In order to permit the most satisfactory interpretation of the factors, the principal axes of the factor matrix were rotated. This rotation which attempts to simplify the columns of a factor matrix has, according to Kaiser (1958), an "invariance property," that is, the factorial description of a test developed using this technique remains to a large extent invariant when the test is moved to another battery containing the same common factors. In the varimax method emphasis is placed upon the simplification of the columns of the factor matrix by maximizing the variance of the squared factor loadings. The components of a factor then tend to unity or zero. The rotated factor matrix for the total sample is shown in table XI.

Interpretation of the Factor Analysis.

In the rotated factor matrix, items having loadings in excess of the arbitrary value .4 were noted, and were considered to be the items which defined a particular factor. From the five subtests, consisting of forty-nine variables, seven factors were isolated. It should be noted that a small number of items did not load highly on any one of the seven factors. These items were therefore excluded from further analysis. Table XII gives a summary of the items discarded as a result of the factor analysis.

TABLE X

UNROTATED FACTOR MATRIX FROM THE ANALYSIS OF ITEMS FIVE TO FIFTY-THREE AND REFERENCE VARIABLES

Variable	Common-Factor Loadings							Commun-ality
	I	II	III	IV	V	VI	VII	
5	179	051	-187	013	101	221	-153	153
6	162	085	-210	141	386	372	056	389
7	208	228	-208	015	313	415	140	429
8	060	415	015	132	407	139	-040	381
9	305	181	-117	032	535	229	344	598
10	271	234	-080	039	381	340	428	583
11	315	515	-332	128	202	-242	-020	592
12	401	534	-314	189	149	-305	-080	705
13	320	525	-255	220	038	-241	-228	604
14	340	372	-285	360	035	-247	-305	620
15	322	219	-273	296	116	-331	-335	550
16	115	054	-235	-265	039	214	-007	189
17	438	196	-042	-401	-164	020	-039	422
18	341	005	-103	-435	-164	233	-205	440
19	229	317	228	-027	-209	-079	042	258
20	440	347	-176	-412	-052	231	-151	596
21	302	417	117	-066	-202	037	337	440
22	394	514	-044	-230	-180	-102	113	532
23	180	393	194	-300	-175	-113	198	398
24	090	-030	031	-223	105	157	-460	306
25	380	372	326	-126	-267	111	119	503
26	046	199	-044	-103	-031	-001	186	090
27	376	253	-281	-367	-142	259	-276	585
28	095	177	074	-097	-148	-183	319	212
29	399	019	-203	-476	-121	189	-216	525
30	259	413	083	-243	-115	-253	154	407
31	087	096	133	-121	-085	012	000	057
32	229	-077	-126	-191	139	146	-072	157
33	670	-308	030	-215	263	-167	065	692
34	662	-310	-005	-121	222	-370	047	739
35	620	-262	033	-090	114	-033	131	495
36	685	-362	-012	-188	062	-038	-053	644
37	603	-314	002	-134	144	-281	018	581
38	626	-279	-067	-173	125	064	006	524
39	741	-194	107	-153	036	-125	-004	640

TABLE X (continued)

Variable	Common-Factor Loadings							Commun-ality
	I	II	III	IV	V	VI	VII	
40	750	-246	095	-072	103	-182	-075	688
41	840	-210	019	-077	054	-094	025	769
42	817	-271	029	-007	106	-071	015	760
43	698	-161	-227	325	-229	174	-026	756
44	666	-076	-014	300	-208	177	065	619
45	690	-077	037	351	-245	191	074	709
46	686	-068	-057	337	-224	157	017	668
47	727	-102	-050	314	-183	154	026	700
48	702	-116	-107	481	-327	215	030	903
49	609	-170	291	080	-160	-041	070	524
50	546	-114	430	126	-070	-037	025	520
51	223	-175	690	146	201	182	-184	687
52	224	213	749	064	263	086	-255	803
53	224	214	748	064	263	083	-256	803
54	271	-021	-071	-124	256	-371	251	360
55	043	059	010	-003	026	-185	-077	046
Sum of Squares	11.063	3.665	3.021	2.669	2.105	2.098	1.645	26.35
% of Common Variance	42.6	13.9	11.4	10.1	8.0	7.9	6.1	100.0

Decimal points have been omitted for factor loadings.

TABLE XI

ROTATED FACTOR MATRIX FROM THE ANALYSIS OF ITEMS FIVE TO FIFTY-THREE AND REFERENCE VARIABLES

Variable	Common-Factor Loadings							Commun-ality
	I	II	III	IV	V	VI	VII	
5	011	098	003	265	-113	203	137	153
6	009	062	014	112	-173	573	114	389
7	-022	032	-015	198	029	614	101	429
8	-113	296	248	015	049	458	-078	381
9	235	057	017	-027	086	729	-001	598
10	107	-052	-008	-027	194	722	087	583
11	093	659	-103	061	234	278	044	592
12	129	745	-065	058	253	210	130	704
13	-008	718	012	115	192	087	173	604
14	-431	608	-088	-124	-165	-075	-096	620
15	159	693	008	051	-090	-023	183	550
16	035	-055	-151	353	041	185	-030	189
17	239	035	-007	447	396	-022	077	422
18	159	-113	-006	610	137	-037	095	440
19	-002	099	184	019	433	-083	141	258
20	106	119	012	645	337	183	081	596
21	004	016	013	-001	616	155	191	440
22	089	252	-031	241	623	067	090	532
23	041	014	071	087	613	-028	-077	398
24	036	039	250	442	-189	-057	-078	306
25	020	-050	266	142	590	014	246	503
26	-019	015	-095	014	257	113	-036	090
27	035	127	-062	708	190	080	138	584
28	082	-011	-104	-143	416	-025	-014	212
29	228	-039	-072	665	136	-009	072	525
30	136	198	015	069	581	-031	-070	407
31	018	-043	103	083	186	-048	005	057
32	208	-036	-014	289	-063	157	013	157
33	805	-010	072	137	026	102	095	692
34	842	132	018	020	020	-017	102	739
35	631	-079	039	089	063	119	251	495
36	701	-072	046	271	-027	-008	265	643
37	743	063	018	069	009	-049	127	581
38	606	-077	014	273	-021	134	239	524
39	693	014	144	190	156	-015	278	640

TABLE XI (continued)

Variable	Common-Factor Loadings							Commun-ality
	I	II	III	IV	V	VI	VII	
40	736	091	175	148	049	-031	285	688
41	743	057	089	192	127	061	384	769
42	744	042	113	142	036	089	401	760
43	315	104	100	161	-038	082	775	756
44	285	026	048	065	103	105	713	619
45	276	009	089	030	119	090	775	709
46	282	086	034	079	077	083	748	668
47	346	072	044	089	065	105	741	700
48	224	074	-022	037	025	068	916	903
49	457	-094	233	-019	198	-098	449	524
50	403	-075	396	-084	181	-071	387	520
51	183	-224	747	-100	-111	013	150	687
52	077	022	879	-029	134	064	001	803
53	078	024	879	-031	133	062	001	803
54	483	161	-123	-145	164	113	-159	360
55	015	149	023	-044	012	-100	-106	046
Sum of Squares	7.295	2.648	2.745	3.007	3.101	2.459	5.108	26.36
% of Common Variance	27.7	10.1	10.4	11.4	11.7	9.3	19.4	100.0

Decimal points have been omitted for factor loadings.

[illegible]

It is clear that those items discarded were of two types, either (a) practically all students were able to handle them or (b) few students could handle them successfully. It may seem strange that items 26 and 31, the recognition of groups of 7 and 8 blocks respectively did not load highly on any factor while item 28, the recognition of groups of 9 blocks did. One explanation might be that those children who really thought about it reasoned that the group of nine blocks must be the largest group and were able to recognize it as a result.

The factors isolated by this method of factor analysis corresponded fairly closely to the subtests as originally set up. There were,

TABLE XII

TEST ITEMS DISCARDED AS A RESULT OF THE FACTOR ANALYSIS

Item Number		Highest Loading
5	Comparison of two equal groups of 7 blocks matched in one-to-one correspondence	.26
16	Recognition of a group of 2 buttons on a card	.35
26	Recognition of a group of 7 blocks in random pattern	.25
31	Recognition of a group of 8 blocks in random pattern	.18
32	Recognition of the first horse in a line of ten horses	.20
49	Select a group of 11 sticks using the bundles of ten and some single sticks	.23

however, two important variations, in that the subtest dealing with cardinal number and the subtest which was concerned with ordinal number were each divided into two factors.

It will be remembered that the subtest on cardinal number was divided into two parts, the recognition of groups ranging in size from 2 to 9 in pattern arrangement and the recognition of groups ranging in size from 2 to 9 in random arrangement. The factor analysis divided this subtest into two factors thus showing that the skills required to recognize groups of three, four and five in pattern arrangement and groups of three and four in random arrangement were independent of the skills required to recognize groups of 6 to 9 in pattern form and groups of 5 to 9 when arranged in random pattern.

Similarly the subtest dealing with ordinal number also turned out as two separate factors. Again it will be recalled that there were two parts to this subtest. In the first part the child was required to find a particular object in a row of objects, for example, the third horse in a row of ten horses, while in the second part he had to give more thought to finding the answer since he was required to do such things as finding the third object in the second row from a group of fifty objects arranged in five rows with ten in each row. It was to be expected that a separate skill was required in each of these two cases, hence the two factors.

Factors Isolated

The various factors isolated by this analysis are described briefly below:--

Factor I -- Ordinal Number, one dimension

This factor was defined by items 33 to 42 inclusive. These items were all concerned with ordinal number in one dimension as shown in the test in Appendix A. It is to be noted that this factor was the biggest contributor to the total variance of the entire sample.

Factor II -- Conservation of Number

This factor was defined by items 11 to 15 inclusive which attempted to measure the child's understanding of the invariance of number (see Appendix A).

Factor III -- Place Value

This factor was defined by items 50 to 53 which were concerned with the child's understanding of the construction of the decimal system of notation.

Factor IV -- Group Recognition (to groups of five inclusive)

This factor was defined by items 17, 18, 20, 24 and 27 which attempted to measure the child's ability to recognize groups of 3, 4 and 5 in pattern arrangement and groups of 3 and 4 in random arrangement.

Factor V -- Group Recognition (groups of five or more)

This factor was defined by items 19, 21, 22, 23, 25, 28 and 30 and attempted to measure the child's ability to recognize groups of 6 to 9 in pattern arrangement and groups of 5 to 9 in random arrangement.

Factor VI -- Comparisons

This factor was defined by items 6 to 10 which attempted to determine the child's ability to make comparison among equal and unequal groups of objects.

Factor VII -- Ordinal Number, two dimension

This factor was defined by items 43 to 48 inclusive and attempted to determine the child's facility with ordinal number.

Table XIII provides a list of the numbers of the items defining the various factors, together with the loading of the item on the factors in question. The reader is asked to refer to the test in Appendix A to determine the exact nature of the items.

On the basis of those items defining the various factors, factor scores were computed for each child in the manner described previously for scoring the test. All further analysis will therefore be based on the factor scores, rather than on subtest scores. Table XIV presents a summary of the factor scores for the entire sample.

It would appear from a study of table XIV that the performance of these children on the skills being measured by Factor IV -- Group Recognition, up to groups of five, and Factor VI -- Comparisons, was better than that on any of the other factors. Perhaps this is attributable to the fact that many of the activities engaged in at playschool help to provide a background for these concepts.

The concept of place value (Factor III) as measured by this test appears to be the most difficult for this group of children. The other concepts are all possessed by these children in varying degrees of accuracy.

The Relationship of Counting Scores with Factor Scores[3]

As noted earlier, counting scores were arrived at by assigning

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[3]Factor scores for the total sample are presented in table XXX in Appendix C.

TABLE XIII

ITEMS DEFINING FACTORS

Item Number	Factors						
	I	II	III	IV	V	VI	VII
5							
6						57	
7						61	
8						45	
9						72	
10						72	
11		65					
12		74					
13		71					
14		60					
15		69					
16							
17				44			
18				61			
19					43		
20				64			
21					61		
22					62		
23					61		
24							
25					59		
26							
27				70			
28					41		
29				66			
30					58		
31							
32							
33	80						
34	84						
35	63						
36	70						
37	74						
38	60						
39	69						

TABLE XIII (continued)

Item Number	Factors						
	I	II	III	IV	V	VI	VII
40	73						
41	74						
42	74						
43							77
44							71
45							77
46							74
47							74
48							91
49							
50			39				
51			74				
52			87				
53			87				

Decimal points have been omitted for factor loadings.

TABLE XIV

SUMMARY OF FACTOR SCORES FOR THE TOTAL SAMPLE
(N = 124)

Factor		Possible Score	Mean	Variance
I	Ordinal Number (one dimension)	10	3.46	12.5
II	Conservation of Number	5	1.79	3.0
III	Place Value	4	0.38	1.0
IV	Group Recognition (up to 5)	5	3.13	2.6
V	Group Recognition (above 5)	7	1.26	2.7
VI	Comparisons	5	3.60	2.1
VII	Ordinal Number (two dimension)	6	1.23	4.8

logarithmic (base 10) values to the highest number reached in each item on counting. These values were then multiplied by ten and summed for all four items in the subtest to arrive at a composite counting score. In an attempt to determine the relationship between counting and the underlying concepts of number, the intercorrelations of counting scores with factor scores were calculated for each of three age groups, five to five and a half years, five and a half to six years and six years and over. These intercorrelations are presented in table XV.

A study of table XV indicates that in the first age group, five to five and a half years, the relationship of counting to ordinal number,

conservation of number, cardinal number and comparisons is significant at the .01 level of confidence. The relationship of counting to a knowledge of place value as measured by this test is not significant at this stage.

TABLE XV

INTERCORRELATIONS OF COUNTING SCORES WITH FACTOR SCORES FOR THREE AGE GROUPS, FIVE TO FIVE AND A HALF YEARS, FIVE AND A HALF TO SIX YEARS AND SIX TO SIX AND A HALF YEARS
($N_1 = 49$, $N_2 = 40$, $N_3 = 35$)

Factors		Group 1 $r \geq .354$ at .01 level	Group 2 $r \geq .393$ at .01 level	Group 3 $r \geq .418$ at .01 level
I	Ordinal Number (one dimension)	.41*	.49*	.72*
II	Conservation of Number	.52*	.13	.11
III	Place Value	.23	.61*	.44*
IV	Group Recognition (up to 5)	.40*	.23	.19
V	Group Recognition (above 5)	.48*	.35	.25
VI	Comparisons	.38*	.16	.21
VII	Ordinal Number (two dimension)	.39*	.23	.54*

*Significant

In the second age group, five and a half to six years, there is a significant relationship between the child's ability to count and his knowledge of the concepts of ordinal number in one dimension, and place value. The relationship of counting to the child's knowledge of conservation of number, cardinal number, comparisons and ordinal number in two dimensions was not significant at this stage.

For group three, age six years and over the relationship of the child's counting ability to ordinal number both in one and two dimensions and place value, was significant. The relationship of counting to all other concepts which these factors were attempting to measure were not significant at the .01 level of confidence.

IV. DISCRIMINANT ANALYSIS

Attempts have been made in recent years to generalize the univariate analysis of variance technique to the case of multiple variates. The extension of the theory has been slow, and only a few methods have been made available for practical use. One of these methods was initiated by Fisher (1936) with his introduction of the discriminant function analysis. By this analysis a set of multiple measurements may be used to provide a linear discriminant function in the observations having the property that, better than any other linear function, it will discriminate between any two chosen classes such as the sexes, and so forth.

The introduction of the discriminant function led to a new method of deriving test criteria suitable for multiple variates. The problem is reduced to that of finding a linear compound of several variables,

where the compounding coefficients are chosen to maximize the value of a statistic suitable for a single variate (see Rao 1952, Chapters 7 and 8).

The stepwise discriminant analysis procedure, used in this study in an attempt to discriminate between the various subgroups of the sample, was such that the variables were weighted according to their power as discriminants, and the ratio of the mean square due to regression to the error mean square was calculated, thus providing an F ratio. At successive steps in the analysis the least powerful variable was deleted and subsequent F ratios calculated. This procedure was carried out in seven steps, leaving only the most powerful variable remaining as a discriminant. Thus the variables represented by the seven factors were ordered in their importance to act as discriminants between the various subgroups selected.

This statistical procedure was used in the making of comparisons between the following subgroups:--

1. Boys and girls in the age group up to and including five and a half years.
2. Boys and girls in the age group between five and a half and six years (not including six years).
3. Boys and girls in the age group six years and over.
4. Children of high socio-economic status and those of low socio-economic status.
5. Children who have older brothers and sisters attending school and those who have not.
6. Children in the age group five to five and a half years and those

between five and a half and six years.

7. Children in the age group six years and over and those between five and six years.

The results of this analysis will now be presented.

Effect of Sex on the Child's Concept of Number

Many studies have shown that girls are superior to boys in their number knowledge. To determine whether this was so in this investigation, with the sample described previously and using the factor scores derived from the test used in this study, the stepwise discriminant analysis procedure, described above, was used. Three age groups were selected as follows:--

Group 1 -- Children in the age group up to and including five and a half years.

Group 2 -- Children in the age group five and a half to six years, but not including six years.

Group 3 -- Children of six years and over.

A comparison of boys and girls was made for each of these age groups. The results of this analysis are presented in tables XVI to XVIII.

A study of table XVI reveals that there is no significant difference at the .01 level in the number concepts of boys and girls in the age group up to and including five and a half years. Any and all combinations of the variables as shown in the table fail to reveal any differences that are significant at this age level.

Table XVII shows the comparison of boys and girls in the second age group, between five and a half and six years. Again the differences

TABLE XVI

COMPARISON OF BOYS AND GIRLS IN THE AGE GROUP FIVE TO FIVE AND A HALF YEARS, BY STEPWISE DISCRIMINANT ANALYSIS
(N_1 = 25, N_2 = 24)

Variables Selected	Coefficients for each Variable											
	I	II	III	IV	V	VI	VII	$\bar{Z}_1$	$\bar{Z}_2$	M.S.R.	M.S.E.	F Ratio
I II III IV V VI VII	031	008	-020	-034	082	-034	-013	-018	-090	124	277	0.44*
I III IV V VI VII	028		-022	-033	086	-032	-009	-017	-088	144	270	0.53*
I III IV V VI	024		-021	-035	088	-032		-025	-095	171	264	0.64*
I IV V VI	022			-032	080	-033		-029	-098	211	259	0.81*
I V VI	018				069	-035		027	-083	245	255	0.95*
I V	016				067			129	081	299	253	1.18*
V					039			051	029	487	250	1.94*

*Not significant

NOTE: Decimal points have been omitted.
Variables refer to factor scores.
Coefficients refer to the optimum weight of each variable as a discriminant function.
$\bar{Z}_1$ and $\bar{Z}_2$ refer to the weighted means of group 1 and 2 respectively.
M.S.R. refers to the mean square due to regression.
M.S.E. refers to the error mean square.

TABLE XVII

COMPARISON OF BOYS AND GIRLS IN THE AGE GROUP FIVE AND A HALF TO SIX YEARS, BY STEPWISE DISCRIMINANT ANALYSIS
($N_1 = 20$, $N_2 = 20$)

Variables Selected	Coefficients for each Variable											
	I	II	III	IV	V	VI	VII	$\bar{Z}_1$	$\bar{Z}_2$	M.S.R.	M.S.E.	F Ratio
I II III IV V VI VII	-022	-022	-011	-034	-009	029	-003	-081	-142	087	293	0.29*
I II III IV V VI	-022	-023	-016	-034	-009	029		-082	-143	101	284	0.35*
I II IV V VI	-023	-022		-034	-010	028		-085	-146	121	276	0.44*
I II IV VI	-022	-027		-037		029		-090	-150	150	268	0.55*
I II IV	-024	-019		-033				-174	-229	181	262	0.69*
I IV	-026			-033				-149	-200	256	256	1.00*
I	-032							-016	-029	401	252	1.59*

*Not significant

NOTE: Decimal points have been omitted.
Variables refer to factor scores.
Coefficients refer to the optimum weight of each variable as a discriminant function.
$\bar{Z}_1$ and $\bar{Z}_2$ refer to the weighted means of group 1 and 2 respectively.
M.S.R. refers to the mean square due to regression.
M.S.E. refers to the error mean square.

TABLE XVIII

COMPARISON OF BOYS AND GIRLS IN THE AGE GROUP SIX TO SIX AND A HALF YEARS, BY STEPWISE DISCRIMINANT ANALYSIS
(N_1 = 18, N_2 = 17)

Variables Selected	Coefficients for each Variable													
	I	II	III	IV	V	VI	VII	$\bar{Z}_1$	$\bar{Z}_2$	M.S.R.	M.S.E.	F Ratio
I II III IV V VI VII	008	034	021	-146	056	-049	-031	-503	-691	235	262	0.89*
II III IV V VI VII		034	025	-136	053	-055	-026	-526	-713	271	254	1.06*
II IV V VI VII		035		-137	059	-057	-022	-539	-722	320	246	1.29*
II IV V VI		029		-124	047	-073		-545	-717	378	240	1.56*
IV V VI				-113	044	-069		-568	-733	478	235	2.03*
IV VI				-080		-076		-542	-688	641	233	2.75*
IV				-126				-216	-303	1071	232	4.61*

*Not significant

NOTE: Decimal points have been omitted.
Variables refer to factor scores.
Coefficients refer to the optimum weight of each variable as a discriminant function.
$\bar{Z}_1$ and $\bar{Z}_2$ refer to the weighted means of group 1 and 2 respectively.
M.S.R. refers to the mean square due to regression.
M.S.E. refers to the error mean square.

between the sexes are not statistically significant at the .01 level.

The comparison of boys and girls of six years of age and over as shown in table XVIII reveals a similar finding. The differences between the sexes at this age level are not significant.

Effect of Socio-Economic Status on the Child's Concept of Number

In an attempt to determine what effect, if any, socio-economic status has on the child's concept of number, the total sample was divided into two groups. The two groups were designated as high socio-economic status and low socio-economic status. The procedures followed in categorizing children of the sample into two groups have been described earlier. Discriminant function analysis was again used in this attempt to discriminate between the two groups specified. The results of this analysis are shown in table XIX.

A study of table XIX reveals that the difference between these two groups is highly significant. Factor 1 -- ordinal number, in one dimension, proved to be the most effective variable in discriminating between these two groups, although every combination of the variables shown in the table reveals differences which are highly significant.

Effect of Siblings on the Child's Concept of Number

What effect does older brothers and sisters who are attending school have on the pre-school child's concept of number? In an attempt to determine the effect, if any, which siblings have on the child's concept of number, a comparison was made between those children who have older brothers and/or sisters attending school and those who have not. The results of this analysis are shown in table XX.

TABLE XIX

COMPARISONS OF CHILDREN OF HIGH AND LOW SOCIO-ECONOMIC STATUS
BY STEPWISE DISCRIMINANT ANALYSIS
($N_1 = 57$, $N_2 = 67$)

Variables Selected							Coefficients for Each Variable											
							I	II	III	IV	V	VI	VII	$\bar{Z}_1$	$\bar{Z}_2$	M.S.R.	M.S.E.	F Ratio
I	II	III	IV	V	VI	VII	033	064	015	056	071	007	022	745	301	1953	147	13.23*
I	II	III	IV	V		VII	033	062	015	055	071		022	764	320	2276	146	15.54*
I	II		IV	V		VII	034	063		054	074		024	763	320	2727	145	18.75*
I	II		IV	V			041	068		056	074			772	337	3354	146	22.96*
I	II			V			049	075			092			645	234	4214	151	27.85*
I				V			056				113			528	177	5403	165	32.70*
I							066							148	026	6831	196	34.77*

*Significant

NOTE: Decimal points have been omitted.
Variables refer to factor scores.
Coefficients refer to the optimum weight of each variable as a discriminant function.
$\bar{Z}_1$ and $\bar{Z}_2$ refer to the weighted means of group 1 and 2 respectively.
M.S.R. refers to the mean square due to regression
M.S.E. refers to the error mean square.

TABLE XX

COMPARISON OF CHILDREN WHO HAVE OLDER BROTHERS AND/OR SISTERS ATTENDING SCHOOL WITH THOSE WHO HAVE NOT, BY STEPWISE DISCRIMINANT ANALYSIS
(N_1 = 84, N_2 = 40)

Variables Selected							Coefficients for each Variable											
							I	II	III	IV	V	VI	VII	$\bar{Z}_1$	$\bar{Z}_2$	M.S.R.	M.S.E.	F Ratio
I	II	III	IV	V	VI	VII	014	033	032	010	014	-028	-024	054	013	159	223	0.71*
I	II	III		V	VI	VII	015	033	029		018	-026	-024	039	-002	182	222	0.82*
I	II			V	VI	VII	017	031			024	-025	-021	043	005	204	220	0.93*
I	II			V		VII	017	025			023		-023	117	084	219	220	0.99*
I	II					VII	018	031					-022	101	075	242	219	1.10*
I	II						011	026						091	071	269	219	1.22*
	II							031						060	046	363	219	1.66*

*Not significant

NOTE: Decimal points have been omitted.
Variables refer to factor scores.
Coefficients refer to the optimum weight of each variable as a discriminant function.
$\bar{Z}_1$ and $\bar{Z}_2$ refer to the weighted means of group 1 and 2 respectively.
M.S.R. refers to the mean square due to regression.
M.S.E. refers to the error mean square.

The differences between these two groups of children were not significant at the .01 level. It should be noted, however, that the comparison was made between all children who had older brothers and sisters attending school and those who did not. Perhaps if the comparison had been made between those pre-school children who have brothers and/or sisters in the primary or early elementary grades, with those who have not, the results might have been different.

Effect of Age on the Child's Concept of Number

The age of children in the sample ranged from five years four months to six years five months. Does this difference in age have any effect on the pre-school child's concept of number? To attempt to answer this question, the sample was again divided into three age groups as follows:--

Group 1 -- Children up to and including five years of age.

Group 2 -- Children between five and a half and six years of age.

Group 3 -- Children six years of age and over.

Since no computer program was available that would carry out a stepwise discriminant analysis for three groups, it was decided to make the comparison by first of all comparing group 1 with group 2 and then comparing group 3 with the first two groups combined.

The results of these analyses are presented in tables XXI and XXII.

A study of table XXI shows that the difference between the first two age groups is not statistically significant.

Table XXII reveals an interesting finding in that there is a

TABLE XXI

COMPARISON OF CHILDREN IN THE AGE GROUP FIVE TO FIVE AND A HALF YEARS WITH THOSE IN THE AGE GROUP FIVE AND A HALF TO SIX YEARS, BY STEPWISE DISCRIMINANT ANALYSIS ($N_1 = 49$, $N_2 = 40$)

Variables Selected	Coefficients for each Variable											
	I	II	III	IV	V	VI	VII	$\bar{Z}_1$	$\bar{Z}_2$	M.S.R.	M.S.E.	F Ratio
I II III IV V VI VII	-035	-025	081	-030	-002	-048	062	-272	-370	309	245	1.26*
I II III IV VI VII	-035	-030	079	-031		-048	063	-272	-370	360	242	1.48*
I III IV VI VII	-033		082	-034		-058	055	-275	-364	349	241	1.63*
I III VI VII	-038		089			-063	051	-210	-288	429	242	1.77*
I VI VII	-032					-059	056	-205	-267	456	243	1.88*
VI VII						-059	026	-166	-202	395	246	1.60*
VI						-053		-172	-197	570	246	2.31*

*Not significant

NOTE: Decimal points have been omitted.
Variables refer to factor scores.
Coefficients refer to the optimum weight of each variable as a discriminant function.
$\bar{Z}_1$ and $\bar{Z}_2$ refer to the weighted means of group 1 and 2 respectively.
M.S.R. refers to the mean square due to regression.
M.S.E. refers to the error mean square.

TABLE XXII

COMPARISON OF CHILDREN IN THE AGE GROUP FIVE TO SIX YEARS, WITH THOSE IN THE AGE GROUP SIX TO SIX AND A HALF YEARS, BY STEPWISE DISCRIMINANT ANALYSIS
($N_1 = 89$, $N_2 = 35$)

Variables Selected	Coefficients for each variable											
	I	II	III	IV	V	VI	VII	$\bar{Z}_1$	$\bar{Z}_2$	M.S.R.	M.S.E.	F Ratio
I II III IV V VI VII	-048	-033	-064	-042	024	-004	021	-271	-518	886	160	5.43*
I II III IV V VII	-048	-034	-064	-043	025		020	-259	-506	1034	162	6.39*
I II III IV VII	-049	-029	-048	-034			019	-256	-498	1213	161	7.51*
I II III IV	-043	-025	-040	-033				-250	-486	1484	161	9.21*
I II IV	-048	-026		-032				-253	-482	1926	161	11.94*
I IV	-050			-038				-233	-454	2772	162	17.13*
I	-057							-050	-117	5111	164	31.15*

*Significant

NOTE: Decimal points have been omitted.
Variables refer to factor scores.
Coefficients refer to the optimum weight of each variable as a discriminant function.
$\bar{Z}_1$ and $\bar{Z}_2$ refer to the weighted means of group 1 and 2 respectively.
M.S.R. refers to the mean square due to regression.
M.S.E. refers to the error mean square.

highly significant difference in the number concepts of children of six years of age and over and those between the ages of five and six years. The most effective discriminant for these age groups proved to be factor I -- Ordinal Number in one dimension, though all combinations of the variables, as shown in the table reveal differences that are highly significant.

This latter finding was further substantiated by combining these age groups in another way. Using the same statistical procedures group 1 was compared with group 3 and group 2 was compared with group 3. Again the differences between group 3, the upper age group, and either of the other two were highly significant. Since this analysis was undertaken merely to verify the findings reported above, it will not be reported in more detail.

The information matrices for stepwise discriminant analysis for each of the combinations of groups reported are presented in tables XXIII to XXIX in Appendix B.

SUMMARY OF FINDINGS

The principal findings of this study will be summarized briefly below.

Findings related to hypotheses

Hypotheses 1. There is no significant relationship between the child's verbalizations of the number names and his knowledge of the underlying concepts involved.

This hypothesis was tested by finding the intercorrelation of

counting scores with factor scores for each of three age groups.

For the age group five to five and a half years, the relationship of counting to the child's knowledge of the concepts of ordinal number, cardinal number (group recognition), conservation of number and comparisons was found to be significant, although in no case were the correlations high. The relationship of counting to a knowledge of place value in the decimal system of notation was not significant for this age group.

For the second and third age groups, five and a half to six years, and six to six and a half years, no significant relationship was found between the child's ability to count and his concept of conservation of number, cardinal number or comparisons. The relationship between counting ability and the concepts of ordinal number, and place value was significant for both groups.

Hypothesis 2. There is no significant difference in the number concepts of boys and girls.

This hypothesis was tested for each of three age groups, five to five and a half years, five and a half to six years and six to six and a half years.

The hypothesis was accepted for all age groups, the differences were not found to be significant.

Hypothesis 3. There is no significant difference in the number concepts of children of high socio-economic status and those of low socio-economic status.

This hypothesis was rejected. The differences, favouring children of high socio-economic status were found to be highly significant.

Hypothesis 4. There is no significant difference in the number concepts of children with brothers and/or sisters attending school and those who have no brothers and/or sisters attending school.

This hypothesis was accepted, the differences were not found to be significant.

Hypothesis 5. There is no significant difference in the number concepts of children in the age groups, five to five and a half years, five and a half to six years and six to six and a half years.

No significant differences were observed between the two lower age groups. However the differences between the upper age group and the lower two age groups combined were found to be highly significant. A further comparison of the first age group with the third, and the second with the third also supported this finding.

According to the evidence collected in this study pre-school children who have reached the age of six years or over have a much greater knowledge of the concept of number than do those who are between the ages of five and six years.

Other Findings

1. The child's ability to count rationally by one's is superior to that of counting by two's, five's or ten's.

2. Almost three fourths(72.6%) of the total children in the sample were unable to respond to items involving counting by five's or ten's.

3. The relationship of counting to a knowledge of cardinal number and the conservation of number was found to decrease with age, this relationship being more significant in the younger children than in the

older ones in the sample tested.

4. The relationship of counting to the child's knowledge of ordinal number and place value was found to increase with age, this relationship being much more significant in older pre-school children than in younger ones.

5. The pre-school child's knowledge of crude comparisons, that is comparing two groups of objects to determine whether one group was either equal to, greater than or less than the other, was found to be superior to that of any other concept tested.

6. About four fifths of the children tested showed an almost complete lack of knowledge of the invariance of number. They tended to confuse the number of objects in a group with the spatial arrangement of the objects, thus indicating that their idea of number was being determined by their perceptions. Only about one in five of the children had reached the stage where they could distinguish number from the physical attributes of the objects involved.

7. With cardinal number as determined by the child's ability to recognize groups of objects without counting, it was found that children could recognize groups of up to five in pattern arrangement and up to four in random arrangement. However, the recognition of groups of larger size was found to require a different type of skill. This would seem to indicate that these children had not reached the stage, in their knowledge of cardinal number, where they could identify larger groups through the relationships of the elemental groups included in the larger group.

8. The child's knowledge of the concept of ordinal number revealed wide

variation, and in relatively few instances did these children show any ability to reason with it. When asked to point out the fifth object in a group, and then the third, for example, there was a tendency for these children to point to the sixth object rather than the third. They appeared to be very uncertain of the concept involved.

9. Children generally showed an almost complete lack of knowledge of the construction of our numeration system.

10. As noted earlier, children of high socio-economic status were found to be superior in their number knowledge to those of low socio-economic status and the factor which appeared to contribute most to this difference was that of the concept of ordinal number.

11. Children of six years of age and over were found to be superior to those below six years of age in their knowledge of the number concepts tested, and again the one factor which appeared to be the greatest contributor to this difference was that of the concept of ordinal number.

12. Two stages in the development of the concepts of number were observed among the children tested. At the first stage were those children who were almost completely lacking in their knowledge of specific concepts, while at the second stage were those who displayed what might be termed "intuitive" ideas of number. These two stages were not the same for each child in each of the concepts tested. Further clarification of these two stages of development is given in the conclusions listed in Chapter V.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The Twentieth Century is fast becoming the golden age of science. To keep pace with the demands science has made upon it, mathematics has broadened its scope to include new ideas in its fundamental concepts as well as new concepts which have evolved into whole new areas called modern mathematics. This does not mean that we are to discard old mathematics but merely that we look at the present structure of mathematics in Twentieth Century attire.

Modern programs in elementary mathematics are attempting to provide for the sequential and systematic development of the basic concepts of arithmetic and the emphasis with beginning school children has been placed on the development of the basic ideas underlying number. The introduction of symbols and symbol manipulation comes only after the basic concepts which these symbols represent have been established. No assumptions are made that the beginning school child has any of the basic concepts established, and thus these concepts, beginning with very primitive ideas of number, are developed sequentially and systematically.

But young children have had many experiences with number and number concepts before they begin school. To what extent have these experiences facilitated the development of the concepts of number?

The primary purpose of this investigation was to determine to what extent pre-school children have developed some of these basic concepts of number.

EXPERIMENTAL DESIGN

A test was constructed by the investigator which attempted to determine the pre-school child's knowledge of some of the basic concepts of number. It was made up of six subtests designed to measure the child's concepts of each of the following:--

1. Rational counting.
2. Comparisons -- group relations.
3. Conservation of number.
4. Cardinal number.
5. Ordinal number.
6. Place value as applied to the decimal system of notation.

This test was then administered by the investigator to individual students in a sample of one hundred twenty-four pre-school children.

The sample consisted of seven classes of playschool children, randomly selected from one hundred twenty-three of such classes in the City of Edmonton, Alberta, Canada. The age range of children included in the sample was five years four months to six years five months. The total sample was divided almost evenly between boys and girls and was made up of forty-nine children in the age group up to and including five and a half years, forty children in the age group between five and a half and six years and thirty-five children in the age group six years and over.

It was assumed that this sample was representative of all playschool classes in the city of Edmonton and further that the one hundred twenty-four children comprising the sample were representative of all

children attending these playschools. It should be noted that attendance in playschool is preparatory to entrance to regular school the following year, thus all children in the population sampled are eligible to register in first grade classes in regular schools in September 1963. However not all children in the city of Edmonton attend these playschools. For the school term 1962-63 the total registration of these playschools was approximately three thousand. This number represents about fifty per cent of all prospective first grade children expected to register in first grade classes in all schools of this city in September 1963.

Barring any bias that may have resulted from sampling procedure, every precaution was taken in the administration of the tests to make the test situation the same for every child. It should be noted, however, that this study was limited to the population of pre-school children in the city of Edmonton, Alberta, Canada, and further that it was limited to a specific group within that population, namely those who were attending playschools. Although the writer had the assurance of the Director of Playschools, that these schools are open to all children, this still does not rule out the possibility that there may be a selective factor operating. All conclusions are made with these limitations in mind.

The tests were administered during the months of March and April, 1963, after these children had been in attendance at playschool for a period of approximately six months. It should be noted also that, although no formal instruction is offered in these playschools, the activities engaged in by these children are designed to provide a background for certain concepts including the concept of number.

The tests were scored and the results analysed during the period May 1st to August 15, 1963. Much of the analysis was carried out by means of an I.B.M. 1620 computer through the Computer Services of the University of Alberta. In this analysis the following hypotheses were tested:--

Hypothesis 1

There is no significant relationship between the child's verbalizations of the number names and his knowledge of the underlying concepts involved.

Hypothesis 2

There is no significant difference between the number concepts of boys and girls.

Hypothesis 3

There is no significant difference in the number concepts of children of high socio-economic status and those of low socio-economic status.

Hypothesis 4

There is no significant difference in the number concepts of children with brothers and/or sisters attending school and those who have no brothers and/or sisters attending school.

Hypothesis 5

There is no significant difference in the number concepts of children in the age groups, five to five and a half years, five and a half to six years and six to six and a half years.

SUMMARY OF RESULTS, AND CONCLUSIONS

On the basis of the findings of this study a number of conclusions may be drawn:--

1. The relationship of counting to the child's knowledge of some of the basic concepts of number was found to decrease with the age of the child in the sample tested, the relationship being less significant in the older children than in the younger ones. This was particularly true with respect to the child's knowledge of cardinal number and the concept of the invariance of number.

It would appear therefore that the pre-school child's ability to count is not a reliable criterion of the extent to which he has developed a knowledge of the true concept of number.

Number is an abstract concept entirely independent of the symbols used to represent it and completely apart from any of the physical attributes of the objects involved. However, pre-school children in this sample tended to confuse number with the spatial arrangement of the objects in a group, thus indicating that they had not yet acquired the true concept of cardinal number.

2. The concept of ordinal number contributed more to the total common variance, within the sample tested, than did any other concept measured by the test. Wherever differences were found to exist among the various sub-groups tested, it was the one variable that contributed most to these differences. Its relationship to counting was found to increase with the age of the child, whereas the relationship of cardinal number to counting, as noted above, was found to decrease with the age of the

child. Children generally showed much uncertainty with this concept and there were wide variations in their knowledge of it. Few, if any, could be said to have acquired true facility with it. This would seem to indicate that a thorough understanding of cardinal number is necessary before the child can have real facility with ordinal number and before he can really appreciate the significance of the counting process.

3. The children's performance on items dealing with the relationship between groups, that is, the making of crude comparisons between two groups of objects, to determine whether one group was either equal to, greater than, or less than another, was better than that on any other concept measured by the test. The investigator's observations of the activities engaged in by these children in playschool and his study of the program for these schools leads him to the conclusion that these activities foster the development of the concepts of group relations. It would appear therefore that education can facilitate the development of some concepts of number.

4. Children generally showed an almost complete lack of knowledge of the structure of the numeration system. This would seem to indicate that a thorough knowledge of the basic concepts of cardinal and ordinal number and rational counting must be established before children are able to understand the concept of place value as used in the decimal system of notation.

5. No significant differences were found to exist between the number concepts of boys and girls in the sample tested. This finding is in agreement with that of Mascho (1960) in his study of the child's concept

of measurement. It would therefore appear that sex is not a contributing factor to the pre-school child's concept of number and related concepts.

6. Differences in the number concepts of children of high socio-economic status and those of low socio-economic status were found to be highly significant. The differences favoured those children of high socio-economic level. This would seem to indicate that environment is a contributing factor to the child's concept of number. Admittedly it can be argued that children of high socio-economic status may be more intelligent. However, no attempt was made in this study to determine the mental capacity of these children.

7. Children of six years of age or over were found to be superior, in their knowledge of the number concepts tested, to those below six years of age. The differences were found to be highly significant, with Factor I -- Ordinal Number being the most effective contributor to these differences. Thus it would appear that age is an important factor in the pre-school child's concept of number, and the age of six appears to mark the dividing line, since the performance of children over six years was significantly superior to that of those below this age. However, it can in no way be inferred from this conclusion that children over six years have firmly established the basic concepts. On the contrary, these children continue to show wide variations in their number knowledge and there are numerous gaps which must be filled before these children can be said to have acquired an adequate understanding of these basic ideas of number.

8. Two stages in the development of the number concepts were observed among the children tested. There were those children who showed an almost complete lack of knowledge of a particular concept, and those who had intuitive ideas of the concept being measured. However these two stages were not the same for each child in all of the concepts tested. A child might be at stage one in one particular concept, showing an almost complete lack of knowledge, whereas in another he might show a fair knowledge of the concept and thus could be said to have reached the second stage of development.

According to Piaget's finding there is a third stage in which children show a firm grasp of these basic concepts of number, but there was little, if any, evidence of this stage in the children tested in this investigation. Perhaps, had the age range been greater this third stage would have shown up. As noted earlier there was a sharp division between children of six years and over and those below six years. It would appear that children of six years of age and over have reached a different stage in their development of the number concepts than those below that age.

IMPLICATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

The findings of this study give rise to several implications for educational practice.

First of all it would appear that any program used in the teaching of arithmetic to beginning school children must identify all aspects of the concept of number, and must attempt to develop these concepts

sequentially and systematically. The young child's knowledge of these basic concepts shows wide variations and has numerous gaps, therefore no assumptions can be made that even the most elementary ideas of number are already in his possession.

Since environment plays such a large part in contributing to the child's concept of number, any program followed must attempt to make up for the deficiencies in environment that may be, in many instances, the cause of his retardation in number and traditionally often caused him to be classified as a slow learner.

Young children tend to confuse spatial relationships with number, therefore they must be given much practice in the manipulation of objects so that they can abstract the true concept of number, apart from the physical properties of the objects involved. A large variety of objects must be provided so that the underlying concept of number will not be confused with the size, shape, color or any other common property of these objects.

If the child's experiences with number are to be designed to create an interest in mathematics and a healthy attitude towards the subject, no attempt should be made to introduce the symbols for number until the child has acquired a thorough understanding of what these symbols represent.

It appears that the concept of cardinal number is basic to an understanding of all other concepts. From this the child can abstract a true understanding of ordinal number and attach true meaning to the counting process. This would appear to be contrary to popular opinion

which seems to be that the concepts of cardinal and ordinal number develop more or less concurrently. The structure of our numeration system can only be taught meaningfully after the child has acquired a thorough understanding of cardinal number.

More recent programs in mathematics for primary and elementary school children are being designed to develop the basic concepts prior to the introduction of the number symbols. Not all teachers are fully aware of just what this new approach to number is designed to do. Many teachers continue to regard the teaching of number as the teaching of number symbols and combinations. Some form of in-service education must be provided so that all teachers of primary school children can be fully enlightened on this approach which is so basic to a thorough understanding of number.

Parents, too, must be informed of this approach, so that they will not get the impression that Johnny is not doing any mathematics just because he is not writing number symbols, and carrying out the addition and subtraction combinations in the traditional way. It must also be seen that teachers and parents are made aware of the vital importance to the child of first acquiring the basic ideas of number before any formal operations with the number symbols are introduced.

This study has been an effort to shed some light on the child's concept of number and has revealed many areas for further study. A few suggestions for further investigations are offered below:--

1. Since there appears to be conflicting evidence on the relationship of sex to the child's concept of number, much more research is needed

to determine the exact nature of this relationship.

2. Many studies have shown that environment is a factor which affects the child's concept of number. What aspects of the child's environment facilitates the development of these concepts?

3. Children tend to confuse the number of objects in a group with their spatial arrangement. To what extent does the size and arrangement of the objects contribute to this confusion?

These and many other aspects related to the child's understanding of the basic ideas of number are fruitful areas for further research.

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APPENDICES

APPENDIX A

A TEST OF THE BASIC CONCEPTS OF NUMBER

Subtest I. Rational Counting

(Materials; 110 counting discs, 100 coloured sticks in bundles of 2's, 100 sticks in bundles of 5's, 100 sticks in bundles of 10's.)

1. Here are a number of coloured discs, see how many you can count.

2. Here are several bundles of sticks. There are two sticks in each bundle. Can you count the sticks two at a time, e.g. 2, 4, --.

3. There are five sticks in each of these bundles. Can you count them five at a time.

4. There are ten sticks in each of these bundles (counting them). Can you count the sticks in all these bundles ten at a time.

Subtest II. Comparisons of Equal and Unequal Groups

(Materials; 30 blocks, 17 toy horses and 17 toy riders, 16 toy dolls and 16 toy rattles.)

5. Arrange 14 blocks of equal size into two groups of seven blocks each, placed in one-to-one correspondence.

Are there as many blocks in this group as in this one here?

6. Arrange a group of eight horses and a group of eight riders in random pattern.

Is there a horse for each rider?

7. Arrange a group of seven dolls and seven rattles in random pattern.

Is there a rattle for each doll?

8. Using blocks of equal size, present the child with a group of four blocks and a group of seven blocks in random pattern.

Are there as many blocks here (indicating the group of seven) as here (indicating the group of four)?

9. Using a group of nine dolls and a group of seven rattles arranged in random pattern, ask the following questions.--

Is there a rattle for each doll?

How do you know?

10. Repeat number 9 with a group of ten horses and a group of nine riders.

Is there a horse for each rider?

How do you know?

Subtest III. Conservation of Number

(Materials; 10 dolls and 10 rattles, 15 red blocks and 15 blue blocks, 2 small cylindrical vessels of equal size and shape, 2 large cylindrical vessels of equal size and shape, one large cylindrical vessel taller but smaller in diameter than the other two larger ones, a bottle of coloured liquid.)

11. Arrange the ten dolls and ten rattles in one-to-one correspondence, making sure that the child is convinced that there is a rattle for each doll. Then move the rattles together in a pile and spread the dolls three inches apart.

Is there a rattle now for each doll?

12. Arrange the objects in question 11 again in one-to-one correspondence so that the child can see that each doll has a rattle. Then move the dolls close together in a pile, and spread the rattles about three

inches apart in a line.

Is there a rattle now for each doll?

13. Present the child with a group of fifteen red blocks, and a group of fifteen blue blocks, of equal size, arranged close together and in one-to-one correspondence. Again make sure that the child sees that there are as many red blocks as blue blocks. Then spread the blue blocks apart, leaving a space approximately equal to the width of one block between them. Are there as many red blocks now as there are blue blocks?

14. Present the child with two uniform containers each containing an equal quantity of coloured liquid. Again make sure that the child is convinced that the quantity of liquid in each container is the same. Pour the contents of one container into a container that is taller and smaller in diameter.

Is there as much liquid in this container (indicating the taller one) as in this one?

15. Pour the liquid back into the standard container again, making sure that the child is convinced that the quantity in the two standard containers is the same. Then pour the contents of one of the standard vessels into two smaller vessels.

Is there as much liquid in these two as in this one?

Subtest IV. Cardinal Number

(Materials; 8 cards of coloured buttons containing groups of 2 to 9, 44 small coloured blocks.)

16. Show me a card which has two buttons on it.

17. Show me a card which has five buttons on it.

18. Show me a card which has three buttons on it.

19. Show me a card which has seven buttons on it.

20. Show me a card with four buttons on it.

21. Show me a card with nine buttons on it.

22. Show me a card with six buttons on it.

23. Show me a card with eight buttons on it.

Present the child with groups of blocks ranging in size from two to nine and arranged in random patterns, both within and among groups.

24. Show me a group which has two blocks in it.

25. Show me a group which has six blocks in it.

26. Show me a group which has seven blocks in it.

27. Show me a group which has four blocks in it.

28. Show me a group which has nine blocks in it.

29. Show me a group which has three blocks in it.

30. Show me a group which has five blocks in it.

31. Show me a group which has eight blocks in it.

Subtest V. Ordinal Number

(Materials; 10 toy horses, a toy ladder and a toy fireman, a simulated parking lot of cardboard, with fifty toy cars.

Arrange the ten horses in single file as if running a race.

32. Touch the horse that is in 1st place.

33. Touch the horse that is in 4th place.

34. Touch the horse that is in 7th place.

35. Touch the horse that is in 5th place.

36. Touch the horse that is in 2nd place.

37. Touch the horse that is in 9th place.

Using the fireman and ladder:--

38. Put the fireman on the 2nd step on the ladder.

39. Put the fireman on the 5th step on the ladder.

40. Put the fireman on the 7th step on the ladder.

41. Put the fireman on the 4th step on the ladder.

42. Put the fireman on the 6th step on the ladder.

Arrange the 50 toy cars on the simulated parking lot in five rows of ten cars each. Suppose you are entering the lot from this side (indicate right side).

43. Touch the 2nd car in the 4th row.

44. Touch the 3rd car in the 2nd row.

45. Touch the 5th car in the 3rd row.

Suppose you were entering the parking lot from this side (indicate left side).

46. Touch the 2nd car in the 4th row.

47. Touch the 3rd car in the 2nd row.

48. Touch the 5th car in the 3rd row.

Subtest VI. Place value in the Decimal System of Notation

(Materials: several bundles of 10 sticks and some single sticks.)

49. Using the bundles of ten sticks and the single sticks make a group of 11 sticks.

50. Using the bundles of ten sticks and the single sticks make a group of 19 sticks.

51. Using the bundles of ten sticks and the single sticks make a group of 25 sticks.

52. Using the bundles of ten sticks and the single sticks make a group of 33 sticks.

53. Using the bundles of ten sticks and the single sticks make a group of 41 sticks.

Edmonton,
April 22, 1963

Dear Parent,

With the permission of the Director of Playschools and the cooperation of the playschool teachers, I have been conducting a study of the number concepts of the children in these schools. This study is being carried out under the direction of a committee from the Division of Graduate Studies of the University of Alberta, of which I am a student.

I ask your cooperation in supplying the information required in the form below. Will you please complete the attached form and return it to your child's playschool teacher on or before Monday April 29, 1963. I assure you that all information will be kept confidential, and nothing in the study will, in any way, reveal the identity of any of the participants.

I thank you, in anticipation of your cooperation.

Yours very truly,

Alec T. Brace

.....

Name of Child Presently Attending Playschool _____

Date of Birth _____ _____ _____
Day Month Year

Number of Brothers _____ Number of sisters _____

Number of Children Attending Regular School _____

Grade of Children in School ____ ____ ____ ____ ____

Father's Name _____

Father's Occupation _____

_____
Signature of Parent

TABLE XXIII

INFORMATION MATRIX FOR STEPWISE DISCRIMINANT ANALYSIS COMPARISON OF BOYS AND GIRLS FIVE YEARS TO FIVE AND A HALF YEARS ($N_1 = 25$, $N_2 = 24$)

Row Number	Variables								
	I	II	III	IV	V	VI	VII	$\bar{X}_1$	$\bar{X}_2$
1	435.96	4.33	48.14	57.88	13.59	25.55	219.00	2.52	1.87
2		149.29	5.86	36.98	35.26	40.59	54.00	1.40	1.33
3			40.00	0.428	33.43	11.57	16.00	0.40	0.16
4				125.63	34.77	8.65	52.00	2.60	2.63
5					99.92	5.51	6.00	1.32	0.75
6						113.31	23.00	3.12	3.37
7							214.00	1.04	0.98

NOTE: Values given above represent the sum of squares and cross products about respective means.

TABLE XXIV

INFORMATION MATRIX FOR STEPWISE DISCRIMINANT ANALYSIS COMPARISON OF BOYS AND GIRLS FIVE AND A HALF TO SIX YEARS OF AGE ($N_1 = 20$, $N_2 = 20$)

Row Number	Variables								
	I	II	III	IV	V	VI	VII	$\bar{X}_1$	$\bar{X}_2$
1	388.97	54.97	27.35	66.67	39.25	12.52	83.52	2.15	3.40
2		98.97	3.35	9.67	51.25	28.52	35.52	1.60	1.95
3			11.10	5.55	6.50	3.60	23.65	0.10	0.20
4				112.77	39.25	17.82	23.82	2.80	3.35
5					105.50	16.75	19.75	1.05	1.45
6						77.97	21.97	3.75	3.70
7							105.97	0.50	0.90

NOTE: Values given above represent the sum of squares and cross products about respective means.

TABLE XXV

INFORMATION MATRIX FOR STEPWISE DISCRIMINANT ANALYSIS COMPARISON OF BOYS AND GIRLS SIX YEARS AND OVER
($N_1 = 18$, $N_2 = 17$)

Row Number	Variables								
	I	II	III	IV	V	VI	VII	$\bar{X}_1$	$\bar{X}_2$
1	400.00	32.00	55.00	56.00	47.00	27.00	15.00	5.66	6.35
2		100.40	7.80	24.20	6.60	19.80	31.20	2.33	2.47
3			59.60	10.40	34.20	5.60	48.40	0.83	0.76
4				66.74	44.80	40.83	22.17	3.44	4.41
5					110.40	22.20	54.80	1.50	1.70
6						61.88	38.11	3.50	4.41
7							235.88	1.72	2.41

NOTE: Values given above represent the sum of squares and cross products about respective means.

TABLE XXVI

INFORMATION MATRIX FOR STEPWISE DISCRIMINANT ANALYSIS COMPARISON OF CHILDREN OF HIGH AND LOW SOCIO-ECONOMIC STATUS
($N_1 = 57$, $N_2 = 67$)

Row Number	Variables								
	I	II	III	IV	V	VI	VII	$\bar{X}_1$	$\bar{X}_2$
1	1546.79	171.95	189.79	283.64	141.37	113.39	556.20	5.26	1.92
2		370.54	52.95	98.35	104.12	103.51	152.29	2.59	1.10
3			152.79	34.64	72.37	33.98	128.20	0.84	0.13
4				339.93	134.61	85.45	143.12	3.96	2.41
5					320.67	53.70	78.77	2.05	0.61
6						263.83	104.09	3.94	3.29
7							614.74	2.24	0.39

NOTE: Values given above represent the sum of squares and cross products about respective means.

TABLE XXVII

INFORMATION MATRIX FOR STEPWISE DISCRIMINANT ANALYSIS COMPARISON OF CHILDREN WHO HAVE BROTHERS AND/OR SISTERS ATTENDING SCHOOL AND THOSE WHO HAVE NOT ($N_1 = 84$, $N_2 = 40$)

Row Number	Variables								
	I	II	III	IV	V	VI	VII	$\bar{X}_1$	$\bar{X}_2$
1	1546.79	171.95	179.93	283.64	143.83	113.98	554.04	3.71	2.92
2		370.54	28.06	98.35	104.91	103.51	143.45	1.93	1.50
3			119.41	308.06	80.22	26.35	105.93	0.45	0.25
4				339.93	133.74	85.48	127.64	3.23	2.92
5					322.21	52.30	81.08	1.39	1.00
6						263.83	94.48	3.51	3.65
7							592.54	1.23	1.17

NOTE: Values given above represent the sum of squares and cross products about respective means.

TABLE XXVIII

INFORMATION MATRIX FOR STEPWISE DISCRIMINANT ANALYSIS COMPARISON OF CHILDREN IN AGE GROUP FIVE TO FIVE AND A HALF YEARS WITH CHILDREN FIVE AND A HALF TO SIX YEARS ($N_1 = 49$, $N_2 = 40$)

Row Number	Variables								
	I	II	III	IV	V	VI	VII	$\bar{X}_1$	$\bar{X}_2$
1	832.11	64.42	73.78	130.37	55.47	44.11	299.06	2.20	2.77
2		252.02	7.98	50.80	88.39	73.42	87.05	1.36	1.77
3			51.50	4.59	39.30	13.78	40.47	0.28	0.15
4				243.12	76.15	31.37	73.02	2.61	3.07
5					206.38	24.47	12.48	1.04	1.25
6						196.11	42.07	3.24	2.75
7							321.64	1.00	0.75

NOTE: Values given above represent the sum of squares and cross products about respective means.

TABLE XXIX

INFORMATION MATRIX FOR STEPWISE DISCRIMINANT ANALYSIS COMPARISON OF CHILDREN BETWEEN FIVE AND SIX YEARS OF AGE AND THOSE ABOVE SIX YEARS ($N_1 = 89$, $N_2 = 35$)

Row Number	Variables								
	I	II	III	IV	V	VI	VII	$\bar{X}_1$	$\bar{X}_2$
1	1546.79	171.95	179.93	283.64	143.83	113.98	554.04	2.46	6.00
2		370.54	28.06	98.35	104.91	103.51	143.45	1.55	2.40
3			119.41	30.80	80.22	26.35	105.93	0.22	0.80
4				339.93	133.74	85.45	127.64	2.82	3.91
5					322.21	52.30	81.08	1.13	1.60
6						263.83	94.48	3.46	3.94
7							592.54	0.87	2.05

NOTE: Values given above represent the sum of squares and cross products about respective means.

TABLE XXX

FACTOR SCORES

Age	Sex	(a)	(b)	Counting score	Factors						
					I	II	III	IV	V	VI	VII
5-4	M	H	1	35.9	9	0	0	1	0	3	2
	M	H	0	21.4	6	2	0	4	2	3	0
	M	L	1	12.6	0	0	0	0	0	3	0
	M	L	1	10.8	1	0	0	5	1	5	1
	M	H	1	26.7	4	3	0	4	0	5	6
	M	L	0	10.0	0	1	0	1	1	1	0
	M	L	1	22.3	0	3	0	5	5	3	0
	M	L	1	06.9	2	0	0	5	1	2	0
	M	H	0	56.8	7	5	0	5	1	5	6
	M	H	1	14.6	2	1	0	5	0	2	0
	M	L	1	14.1	0	0	0	0	0	5	0
	M	L	0	14.6	1	0	0	1	0	1	0
	M	H	1	19.4	0	5	0	3	3	1	0
	M	L	0	40.0	0	0	0	2	1	2	0
	M	L	0	46.7	4	1	0	3	0	4	0
5-5	M	L	1	7.8	2	0	0	1	0	0	0
	M	L	1	12.3	0	0	0	1	0	4	0
	M	H	0	17.7	0	5	3	1	2	5	0
	M	H	0	24.6	3	1	0	3	0	2	4
	M	H	1	68.0	7	3	2	3	3	2	6
	M	H	0	12.0	1	2	0	3	2	5	1
	M	H	1	35.4	7	0	0	1	0	4	0
5-6	M	H	1	70.8	7	0	5	3	6	4	0
	M	H	1	12.8	0	3	0	2	5	4	0
	M	L	0	18.4	0	0	0	3	0	3	0
5-7	M	L	1	15.7	3	0	0	4	1	2	0
	M	H	1	61.0	1	3	0	5	2	5	0
	M	H	1	63.1	6	5	0	4	2	5	6
	M	L	1	32.2	1	1	0	4	0	5	0
5-8	M	L	0	3.0	3	1	0	0	0	2	0

TABLE XXX (continued)

Age	Sex	(a)	(b)	Counting score	Factors						
					I	II	III	IV	V	VI	VII
5-8	M	L	0	29.2	0	0	0	4	0	3	0
	M	L	1	13.2	2	1	0	3	0	2	0
	M	L	1	49.6	6	0	0	4	0	5	0
	M	H	1	69.9	9	5	2	4	5	5	4
5-9	M	L	1	11.9	0	2	0	3	5	3	0
	M	L	0	10.0	1	2	0	0	0	5	0
	M	L	1	12.7	0	5	0	2	0	5	0
	M	L	1	14.6	0	0	0	0	0	0	0
	M	L	1	11.7	6	2	0	0	0	4	0
5-10	M	L	1	10.4	0	1	0	2	2	4	0
	M	L	1	12.5	1	2	0	4	1	4	1
	M	L	1	11.1	2	0	0	1	0	5	0
	M	H	0	10.7	2	0	0	3	0	3	0
	M	L	0	10.0	0	1	0	4	1	4	0
5-11	M	H	1	20.1	0	1	0	5	2	4	0
6-0	M	L	1	27.6	4	3	0	3	0	1	0
	M	H	0	32.5	7	2	0	5	3	5	6
	M	L	1	24.7	9	0	0	5	0	5	0
	M	L	1	14.7	1	0	0	2	0	5	0
6-1	M	H	1	70.4	9	3	2	5	5	5	6
	M	H	0	48.5	9	3	1	5	1	5	0
	M	L	1	53.5	9	0	0	1	0	2	3
	M	H	1	70.8	9	2	5	5	7	4	4
	M	L	1	21.8	1	3	0	1	0	3	0
6-2	M	H	1	40.4	4	4	0	3	1	5	0
	M	L	1	31.9	9	0	0	2	0	0	0
	M	H	1	57.1	9	5	2	5	3	5	6
	M	H	1	20.0	0	4	0	4	0	4	0
	M	H	1	51.9	6	2	2	2	3	2	0
6-3	M	H	1	23.6	9	5	2	5	2	2	0
	M	L	0	14.6	6	5	1	3	1	4	6
6-4	M	L	1	10.4	1	0	0	2	0	2	0
6-5	M	L	0	14.7	0	1	0	4	1	4	0

TABLE XXX (continued)

Age	Sex	(a)	(b)	Counting score	Factors						
					I	II	III	IV	V	VI	VII
5-4	F	H	1	62.5	9	4	1	5	2	4	6
	F	L	0	12.3	0	0	0	0	0	5	0
	F	L	0	13.0	0	0	0	4	2	2	0
	F	L	0	10.0	2	0	0	3	0	2	0
	F	H	1	34.4	5	5	0	4	0	4	5
	F	L	0	10.0	0	0	0	1	0	2	0
	F	L	1	27.5	0	0	0	2	1	5	0
	F	L	0	13.2	0	2	0	4	0	5	0
	F	L	1	66.1	4	0	0	2	1	4	0
	F	H	1	15.0	0	4	0	3	1	4	0
	F	H	0	39.2	2	0	0	3	1	5	0
	F	H	1	27.7	9	0	2	3	1	5	6
	F	H	0	9.5	0	1	0	5	3	1	0
	F	L	1	27.8	0	3	0	5	1	5	0
	F	H	1	10.4	2	0	0	5	0	2	0
5-6	F	L	1	14.6	0	2	0	3	2	3	0
	F	L	0	24.8	3	0	1	2	2	4	0
	F	L	1	10.7	0	3	0	0	0	4	0
	F	L	1	21.7	0	0	0	2	0	2	0
	F	L	1	12.5	0	0	0	1	0	0	0
	F	L	0	25.4	0	3	0	1	0	5	0
	F	L	1	12.3	0	0	0	0	0	1	0
	F	L	0	12.3	0	5	0	2	1	5	0
	F	H	1	59.8	9	0	0	3	0	2	6
5-7	F	L	0	23.6	7	0	0	3	0	5	0
	F	H	1	40.0	1	1	0	5	4	4	0
	F	L	1	10.4	0	0	0	5	0	5	0
	F	H	0	29.3	7	2	2	4	1	5	6
5-8	F	H	1	12.7	9	3	0	5	3	1	1
	F	L	0	20.5	0	1	0	0	0	0	0
	F	L	1	13.0	0	4	0	2	2	5	1
	F	H	1	36.4	0	3	0	4	5	5	0
5-9	F	L	1	24.4	6	2	0	5	0	2	0
	F	H	1	26.7	0	2	0	1	0	3	0
	F	L	0	10.7	2	0	0	2	2	4	0

TABLE XXX (continued)

Age	Sex	(a)	(b)	Counting score	Factors						
					I	II	III	IV	V	VI	VII
5-9	F	H	1	62.3	9	5	0	4	5	5	0
5-10	F	L	0	14.1	0	2	0	2	1	3	0
	F	H	1	14.1	4	0	0	5	0	4	2
5-11	F	H	1	24.6	6	3	0	5	3	3	0
	F	L	1	12.0	6	0	2	4	1	3	4
	F	L	1	11.7	0	2	0	0	0	5	0
	F	H	1	39.8	2	4	0	3	2	5	4
	F	L	1	13.8	0	2	0	4	0	3	0
6-0	F	H	1	27.1	9	3	0	4	0	4	0
	F	L	1	11.7	6	2	0	5	1	5	2
	F	H	0	18.1	9	3	0	3	1	4	6
	F	L	1	46.7	8	5	0	4	0	5	2
	F	H	1	20.1	8	3	0	5	1	5	0
6-1	F	H	0	53.3	9	5	2	4	0	5	6
	F	H	1	30.9	5	2	1	5	4	4	6
	F	H	1	52.7	5	2	2	5	5	5	1
	F	H	1	21.0	0	4	0	5	3	4	0
	F	H	1	66.9	5	4	0	5	2	4	0
	F	H	0	36.1	7	0	0	5	3	5	0
	F	H	0	46.5	9	2	0	5	1	4	1
	F	H	0	19.0	9	2	0	5	4	5	5
6-3	F	H	1	25.2	9	3	1	5	2	5	6
	F	L	1	16.8	0	1	0	1	0	2	0
	F	L	1	69.4	9	0	5	3	0	4	6
	F	H	1	53.2	5	4	2	5	0	5	0
	F	L	0	44.5	1	0	0	5	2	4	0

NOTE: Age in column 1 is given in years and months.
H--in column (a) represents a child from a high socio-economic group.
L--in column (a) represents a child from a low socio-economic group.
1--in column (b) represents a child who has older brothers and sisters attending school.
0--in column (b) represents a child who has no older brothers and sisters attending school.

B29809